Decentralized Topological Mapping for Multi-robot Autonomous Exploration under Low-Bandwidth Communication

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***Abstract*— This paper concerns a mapping framework for multi-robot exploration of underground environments with only very limited communication available. We focus on multi-robot map building and coordination to explore large areas with real-time planning to long distances. The considered communication can broadcast only $100\,\mathrm{B/s}$, and therefore, we propose coordination planning using two terrain models. The first model is a dense 3D map built by each robot individually to identify explorable places and generate detailed plans to avoid untraversable areas. The second model is a global topological map built in a decentralized manner by exchanging tiny $12\,\mathrm{B}$ packets between the robots. The feasibility of the proposed approach has been verified in the real-world autonomous exploration mission and various multi-robot scenarios inspired by a virtual cave circuit of the DARPA Subterranean Challenge while adapting two different decentralized coordination strategies.**

I. Introduction

Robotic exploration is a way to avoid risk to humans in dangerous environments in search missions of unknown areas such as underground space mapping or disaster response scenarios. In particular, we are motivated by the DARPA Subterranean Challenge [1] (SubT), where a team of robots is requested to search subterranean environments of mines, caves, and urban-like underground infrastructure. The mission is to search unknown environments for known artifacts such as survivor, cell phone, or backpack and report their position to the operator with the precision of $5\,\mathrm{m}$. Therefore, the problem combines an exploration of the environment to identify areas where the artifacts can be located and an efficient search strategy to cover possible locations by sensors to recognize the artifacts. This paper focuses on mapping, navigation, and robot coordination using ground robots under very limited communication.

Frontier-based exploration [2] is a commonly used technique to search unknown environments by navigating robots towards the border between known and unknown parts of the environment. Extensions to multi-robot exploration build a dense map based on sensed information from all the robots [3], [4]. Sharing dense maps is beneficial for precise navigation, but it is not suitable for low-bandwidth communication. Therefore, bandwidth-saving methods are studied, such as map compression [5] or usage of topological maps [6], [7], [8] that are suitable for planning to long distances at large-scale environments. However, the existing topological mapping approaches do not fit our requirements for autonomous exploration with decentralized mapping for communication technology based on $868/915\,\mathrm{MHz}$ transceivers capable of broadcasting approximately $100\,\mathrm{B/s}$.

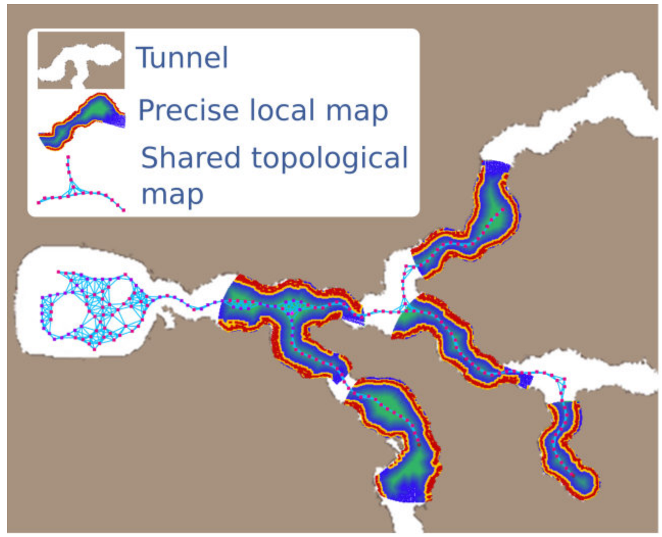

Fig. 1. The topological model shared between the robots is shown with precise local maps created and used by each robot independently.

This paper presents a mapping framework that combines precise local mapping for navigation in uneven terrain with a decentralized topological mapping suitable for coordinated autonomous exploration under low-bandwidth communication. The concept of the proposed hybrid mapping approach is visualized in Fig. 1. The feasibility of the proposed method has been empirically validated, and the evaluation results indicate its suitability to the addressed scenarios with very limited communication bandwidth.

The rest of the paper is organized as follows. A brief overview of the related work focused on environment representation and multi-robot coordination is presented in Section II. The proposed mapping framework and an adaptation of existing exploration strategies into it are described in Section IV. The evaluation results are reported in Section V. Concluding summary is in Section VI.

II. Related Work

Autonomous exploration robots have been deployed in various areas according to which an environment model is selected. A 2D occupancy grid map can sufficiently capture differences between obstacles and free space in office-like environments, and the robot can navigate itself in the environment [4]. More sophisticated methods might be needed

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for exploring uneven [9], [10] and multi-layer terrain [11]. Although a precise global terrain model supports operation in a complex environment, the obvious drawback of traditional occupancy map-based approaches is memory requirements, especially for very large-scale scenarios.

The authors of [12], [13] proposed to address maintenance of a precise global map by a dense local map combined with a sparse topological map. The precise local map primarily serves to detect possible robot-environment collisions and planning short-term paths within the robot's surroundings. Complementary, the sparse topological representation is suitable for planning distant locations outside the area covered by the local map. Such a combination of local dense and sparse topological maps is memory efficient; however, the approaches are targeted to single robot applications.

Other variants of the topological-based methods utilize room detection [14], detection of crossroads and corridor connections [15], or semantic information using behavior-based strategies [16]. In [8], [17], the authors generated topological maps based on the robot motion, which makes the approaches independent of the specific geometrical properties of the space being explored.

Regarding the time needed to explore the environment, a natural way to improve the effectiveness of the exploration is to increase the number of exploring robots. In the early approach [3], the robots share evidence grids, and they are navigated to the closest frontier [2] determined in a global map of the environment. Utility-based assessment of the next-to-visit location has been proposed to improve the exploration performance by evaluating the distance to the goal location and the expected utility to the exploration of not yet covered part of the environment [18], [6]. A rank-based assessment of the next exploration goals called MinPos has been proposed in [19]. In MinPos, possible next navigational locations are ranked using the location distance to the actual robot and the other robots. However, these approaches rely on sharing the occupancy grid maps between the robots, which can have enormous communication bandwidth requirements.

On the other hand, multi-robot coordination in exploration scenarios with low bandwidth communication has been addressed using sets of polygons to encode the geometrical maps [20]. In [5], the authors proposed to use the occupancy Normal Distributions Transform (NDT) as a memory-efficient representation, shareable through a network with the bandwidth of $37.5\,\mathrm{kB/s}$ while, according to the paper, at least $4.1\,\mathrm{kB/s}$ of the bandwidth is utilized. Another way is to neglect the exchange of detailed information used in navigation and share only sparse information required for coordination. Such principle is used in approaches [21], [6], [7], [17], where the authors propose to share topological maps and states of the robots for coordination since each robot builds a detailed map necessary for precise navigation and exploration of new parts of the environment individually by robots themselves. Although these methods provide significantly lower bandwidth requirements than the methods based on dense map sharing, they still not met our requirements on bandwidth usage, especially if several robots are deployed simultaneously. Therefore, we propose a minimalistic topological mapping that is even more bandwidth-efficient yet compatible with existing (aforementioned) coordination strategies, including MinPos and its extensions such as [22].

III. Problem Statement

The addressed problem is decentralized coordination of multi-robot mapping in an exploration mission with low-bandwidth communication limited to broadcasting less than $100\,\mathrm{B/s}$ from each robot. The exploration scenario is motivated by deployments within the DARPA SubT. However, in this paper, we focus on environment exploration, and the artifacts to be searched are considered uniformly distributed in the environment. The mission ends when each robot in the team does not have any exploration goal to find an artifact. Thus, it is not required that each robot has to find all the artifacts. The performance indicator of the coordination strategy is the time needed to explore the environment, and a faster strategy is considered better than a slower strategy.

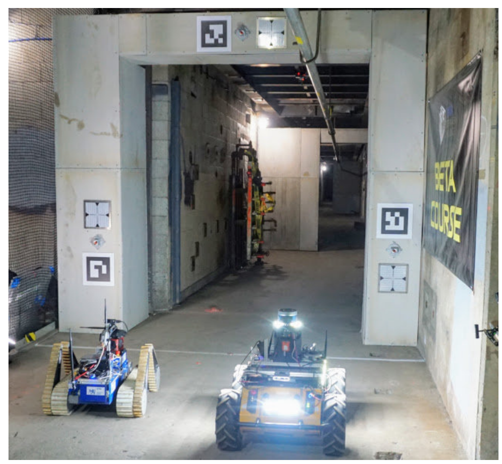

Fig. 2. A gate with markups to initialize the global coordinate frame used in the Urban circuit of the DARPA SubT.

Autonomous multi-robot exploration is a complex problem. Despite individual parts such as mapping, navigation, and coordination being addressed by the herein presented methods, we limit the scope of the paper by the following assumptions.

1) Robots are localized within the same global coordinate frame that is initialized at the beginning of the mission, such as the entry gate at the staging area in the DARPA SubT, see Fig. 2.
2) Robots do not block each other when passing a corridor.
3) Robots operate in an environment with similar speed.

Notice that the last assumption allows us to compare the coordination strategies with less dependence on dynamic models of the particular robots. Assuming the robots operate with similar speed, it holds that the longest traveled path by a robot is proportional to the total mission time. Since the path length does not depend on the computational resources and computational complexity of employed methods, we can fairly compare the results of various methods. Thus, for a given set of L exploration trials, the mean performance of a particular coordination strategy is estimated by the median of longest traveled paths.

$$p_{max} = \text{median}\left(\max\{p_{1,1},\dots,p_{n,1}\},\dots\max\{p_{1,L},\dots,p_{n,L}\}\right), \quad (1)$$

where $p_{i,l}$ is the length of the i-th robot path in the l-th trial, and n is the number of robots.

IV. Proposed Low-Bandwidth Multi-robot Exploration Strategy

The proposed multi-robot exploration is designed under the assumption that communication is unreliable and might not cover the whole environment. Thus temporal losses of connections between the robots can occur. Therefore, we addressed that challenge by designing the whole exploration framework decentralized; thus, a robot can still decide where to explore next, even if it is disconnected from others. Then, the coordinated behavior of the multi-robot team is achieved by running the same exploration strategy on all the robots without relying on any central coordination unit.

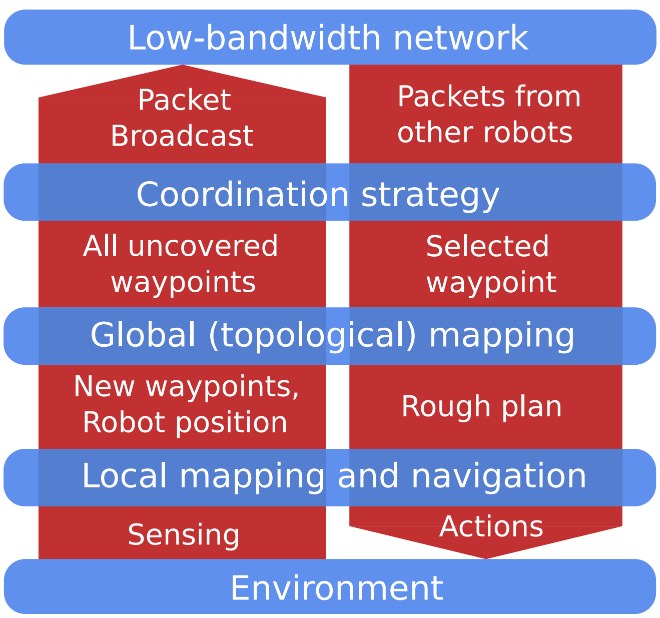

Fig. 3. Overview of the exploration framework and relations between the individual modules.

The exploration framework consists of several modules, and relations between them are schematically visualized in Fig. 3. The main parts of the proposed coordination method based on dense local mapping and topological mapping are detailed in the following parts of this section.

A. Local Mapping and Exploration Waypoints

The local mapping and navigation module creates a dense map to steer the robot through the environment and avoid possible collisions with obstacles and untraversable terrain. The local map is built using 3D LiDAR measurements and 6-DoF robot localization [23], which description is considered to be out of the scope of this paper. The measurements are fused into a 3D point cloud around the robot. However, the map is only local, and points farther than d_{local} from the robot are removed.

An elevation map represented by a 2D grid is extracted from the point cloud to detect traversable parts of the environment. The traversability estimation is based on [24], where a cell ν of the elevation map is considered to be traversable if the maximum difference of its height $h(\nu)$ and height of its neighbors is lower than a threshold h_{diff}, which value is estimated based on the robot kinematics,

$$traversable(\nu) \begin{cases} \texttt{false} & \text{if } \max\limits_{\zeta \in 8nb(\nu)} |h(\nu) - h(\zeta)| > h_{diff} \\ \texttt{true} & \text{otherwise} \end{cases} \quad (2)$$

where $8nb(\nu)$ is the 8-neighborhood of the cell ν and the value of h_{diff} is based on the height of the chassis of the wheeled robot. In particular, $h_{diff} = 13\,\text{cm}$ is used for all herein presented results. The untraversable cells are grown by the radius of the robot shape circumference to consider the physical size of the robot.

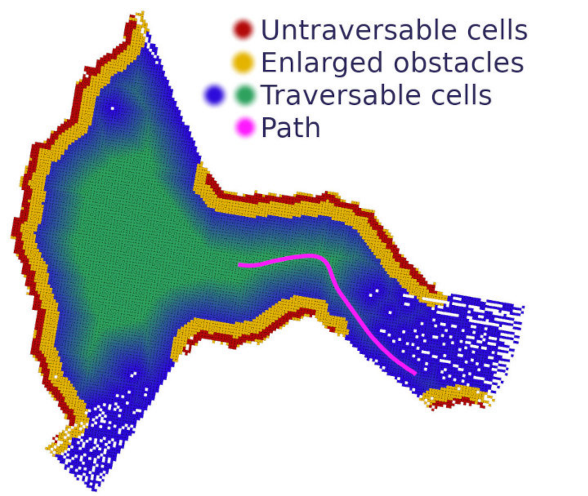

Fig. 4. Example of local map with shown traversable and untraversable cells, where the cell cost is based on the distance from untraversable/unknown space is shown in blue (high) and green (low) colors.

Planning the path from the robot position to the selected waypoint within the local map is done by finding the closest traversable cell to the current robot position and then applying Dijkstra's algorithm [25] to find the shortest path to the waypoint. Similarly to [9], the cost used in planning is computed as the Euclidean distance between the cells increased by the cost related to the distance of cells to the closest untraversable/unknown cell. The applied costs result in the robots tend to pass narrow corridors in the middle. An example of the local map is shown in Fig. 4.

In addition to collision avoidance, the local map is used to identify new possible exploration waypoints $\boldsymbol{W}_k$ merged into the set of the candidate waypoints $\boldsymbol{W}$. The waypoints are generated according to the main mission objective using the sensor coverage model. In our case, we aim to cover the whole environment with the robot's cameras to detect the artifacts. The robot is equipped with cameras providing omnidirectional vision, and we suppose that artifacts are detected if they are closer than d_{cam} to the robot. Thus, for each cell of the locally build grid map, the information gained by observing the cell's surroundings by the camera is approximated by Shannon's entropy of the artifact distribution. We suppose that before the observation, it is equally possible that none or one of a possible types of artifacts can be on a cell. Then, the information gained by observing the surroundings of a cell ν can be expressed as

$$e(\nu) = \sum_{\xi \in \delta(\nu, d_{cam})} \begin{cases} \log(a+1) & \text{if } observable(\nu, \xi) \\ 0 & \text{otherwise} \end{cases}, \quad (3)$$

where $\delta(\nu, d_{cam})$ is the set of all cells ξ such that $\| \nu - \xi \| < d_{cam}$. The function $observable(\nu, \xi)$ returns `true` if the cell ξ is observable from ν determined by ray-casting from ν to ξ in the current local elevation map.

Once the expected information gain of each reachable cell is determined, the possible waypoint locations $\boldsymbol{W}_k$ are selected as a subset of the cells such that the distance between each pair of cells $\boldsymbol{W}$ is at least $w_{cl,radius}$, and the information gain at each waypoint is at least e_{min}. Besides, before merging $\boldsymbol{W}_k$ into $\boldsymbol{W}$, the expected information gain of $\nu \in \boldsymbol{W}$ is updated, and all ν for which $e(\nu) < e_{min}$ are removed from $\boldsymbol{W}$. The waypoint set $\boldsymbol{W}$ is maintained by each robot independently. Based on the shared topological map, a waypoint is removed from $\boldsymbol{W}$ if the other robot indicated its presence closer than d_{cam} to the waypoint.

B. Global Topological Mapping

The global topological map is shared between the robots to support planning and reasoning about long distances. Contrary to the precise local map built by each robot for precise navigation in a short horizon, the topological map enables fast planning in large-scale scenarios. Besides, a low memory footprint of the topological map enables its sharing between the robots even with the used low-bandwidth communication.

The topological map is represented as a graph $\mathbf{T} = (\mathbf{V}, \mathbf{E})$, where $\mathbf{V}$ is a set of vertices, and $\mathbf{E}$ is a set of bidirectional edges. A single vertex $v(\mathbf{p}, ids, e_{inf}) \in \mathbf{V}$ is associated with the 3D position of the robot $\mathbf{p}$, the set of ids indicating the presence of robots that visited the vertex, and estimated information e_{inf} obtainable at the position $\mathbf{p}$. Each edge $\gamma(v_1, v_2) \in \mathbf{E}$ represents a traversable path connecting positions of the incident vertices.

TABLE I

DATA PACKET FOR INFORMATION EXCHANGE.

Data	Sender ID	Position $\mathbf{p}$	Presence of ID at $\mathbf{p}$	e_{inf}
Size	8 bit	72 bit	1 bit	8 bit

The third data field indicates presence of the robot (sender) ID at $\mathbf{p}$. The total size of the packet is rounded to 12 B.

The topological map $\mathbf{T}$ is built from messages generated when the robot is moving through the environment and from messages received from other robots. The message is encoded in the data packet detailed in Table I. A packet is created and shared with other robots when the current robot position is more than d_{build} distant from the position where the previous packet has been generated. Alternatively, when information associated with a certain vertex is changed, i.e., a new waypoint associated with the vertex or waypoint associated with the vertex was covered by the robot. The update of the topological map by a packet is summarized in Algorithm 1.

Algorithm 1: Topological map update by a packet.

Input: $\mathbf{T}(\mathbf{V}, \mathbf{E})$ - topological map, where each $v \in \mathbf{V}$ is associated with the position p, a set of robot $ids[]$, and e_{inf}.
Input: $t(\mathbf{p}, id, p, e_{inf})$ - received packet
Parameters: d_{build}, $d_{connect}$ – minimal and maximal distance d_{build} between the positions associated with the vertices
Output: $T(V, E)$ - updated topological map

```
if  V = ∅ then
    v ← create_vertex_from_packet(t)
    V ← V ∪ {v}                    // Initialize map
else
    h ← argmin_{v∈V} (‖ v.p − t.p ‖)   // Closest vertex
    if ‖ h.p − t.p ‖< d_build then
        h.ids ← v.ids ∪ t.id
                                   // Record visitor's ids
        h.e_inf ← t.e_inf
        V ← replace_vertex(V, h)
    else
        v ← create_vertex_from_packet(t)
        V ← V ∪ v
        connect_edges_to_close_vertices(T, v, d_connect)
return  T(V, E)
```

A new packet indicating a change in the obtainable information at the related position updates the robot's topological map, and the packet is also broadcasted to other robots. Information obtainable at the position $\mathbf{p}$ is computed as

$$e_{inf} = \sum_{i=1}^{p_w} e(w_i), \quad (4)$$

where $e(w_i)$ is information entropy evaluating the i-th of p_w waypoints within the local map that are reachable from $\mathbf{p}$.

Note that generating new waypoints and also discarding waypoints is related to the motion of the robot. Hence, packets indicating a change of the information at a certain vertex and packets related to the motion of the robot at the same vertex are joined into a single packet to save communication bandwidth further.

In selecting the exploration waypoint towards which the robot is navigated next, the traversal cost over long distances is computed by Dijkstra's algorithm using the topological map. When the underlying topological graph is not connected, e.g., due to the communication interruption, a path

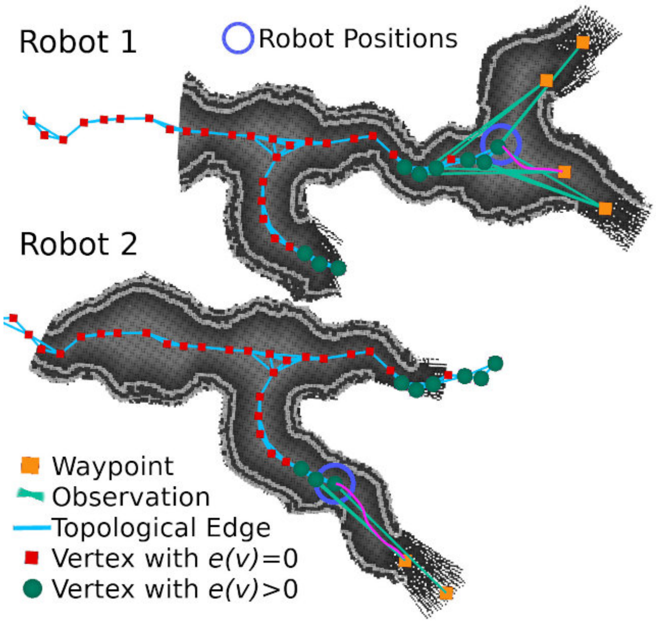

Fig. 5. A visualization of the topological map shared between two robots showed from their perspective. Although the local maps and waypoints are not shared between the robots, the obtainable information saved in the topological map is sufficient to indicate which positions have been reached and whether there is an unobserved part of the environment, i.e., a place where a robot can detect some exploration waypoints while visiting it.

is found from the robot's location to the closest vertex to the waypoint. Since the vertices have associated positions with the expected obtainable information, the topological map is also used to identify unexplored parts of the environment. An example of the visualized topological map with marked vertices with non-zero obtainable information is shown in Fig. 5.

C. Decentralized Coordination Strategy

Coordination strategy is an important part of multi-robot exploration. We adapted two decentralized coordination strategies based on the proposed mapping framework with dense local maps and a shared global topological map. For simplification of the notation, the set $\mathbf{R}$ of all n robots is split into the particular robot r, and the set of the remaining $m = n - 1$ robots $\mathbf{R}' = \mathbf{R} \setminus \{r\}$.

1) Greedy strategy: Our first coordination strategy is based on [3], where each robot is navigated to its closest frontier; thus, we denoted it as **Greedy**. It starts with the identification of all possible targets $\boldsymbol{W}_t$ for the robot r. $\boldsymbol{W}_t$ is a union of all candidate waypoints and positions of the topological vertices V with non-zero obtainable information that have not been visited by r:

$$\begin{aligned} \boldsymbol{W}_t &= \boldsymbol{W} \cup \boldsymbol{W}_{inf}, \\ \boldsymbol{W}_{inf} &= \{\nu \in V, \nu.e_{inf} > 0 \wedge r.id \cap \nu.ids = \emptyset\} \end{aligned} . \tag{5}$$

The next navigational goal is selected as the closest target to the current position of the robot r. The distance to the target $t \in \boldsymbol{W}_t$ is determined as the sum of the shortest path lengths in the topological map and the Euclidean distance between the position $\mathbf{p}_t$ of the target t and the position $\mathbf{p}_v$ of the closest topological vertex.

2) MinPos on the topological map: The second coordination strategy is denoted **MinPos** because it is based on approach described in [19]. **MinPos** ranks all targets $\boldsymbol{W}_t$ obtained for each robot using the same method utilized in the **Greedy** strategy. The robot is then navigated towards the target with the lowest rank. The ranks count how many of the robots $\mathbf{R}'$ have a shorter path to the j-th target than the actual robot r

$$r^{\text{MinPos}}(w_j) = \sum_{\forall R_k \in \mathbf{R}', C_{k,j} < C_{r,j}} 1, \tag{6}$$

where $C_{k,j}$ is the length of the k-th robot path to the j-th target within the topological map. If there are multiple targets with the lowest rank, these targets are compared by **Greedy** strategy to select the next navigational goal.

V. Results

The proposed exploration framework has been implemented in C++ using ROS [26] and verified in two ways. First, the autonomous part combining the precise local navigation with global topological mapping has been tested in a single-robot real-world scenario. Next, the proposed topological mapping with low-bandwidth communication is compared to the standard mapping technique with exchange dense local maps. Computationally efficient S.T.D.R. Simulator [27] has been utilized for the evaluation in 120 simulation trials of autonomous exploration missions.

The exploration framework generates plans for the controlled wheeled robot. An independent process executes plans as sequences of waypoints, and the robot is steered towards the closest waypoint until it is closer than $0.3\,\mathrm{m}$ from the waypoint. Then, the waypoint is removed from the sequence, and the robot starts to follow the next waypoint. The robot steering is realized using forward, and angular velocity commands.

A. Experimental Validation of the Local Mapping

The experimental deployment has been done on the wheeled robot Husky in the environment shown in Fig. 6. During the experiment, all the computations run on the onboard Intel NUC with the Intel i7-8559U CPU clocked at $2.7\,\mathrm{GHz}$ with $8\,\mathrm{GB}$ RAM and 3D LiDAR Ouster S0-128. The used parametrization is depicted in Table II.

TABLE II
Parametrization of the Exploration Framework.

Parameter	Real-world experiment	Simulated scenarios
Resolution	$7.5\,\mathrm{cm}$	$10\,\mathrm{cm}$
Local map size d_{local}	$25\,\mathrm{m}$	$20\,\mathrm{m}$
Distance between vertices d_{build}	$0.4\,\mathrm{m}$	$1.0\,\mathrm{m}$
Connect vertices distance $d_{connect}$	$1.0\,\mathrm{m}$	$2.5\,\mathrm{m}$
3D sensor range	$55\,\mathrm{m}$	$10\,\mathrm{m}$
Omni-camera range d_{cam}	$9.5\,\mathrm{m}$	$9.5\,\mathrm{m}$
Waypoint cluster radius $w_{cl,radius}$	$2.0\,\mathrm{m}$	$3.0\,\mathrm{m}$
Min cluster entropy e_{min}	78 bit	90 bit

Five artifacts of different type were considered in all the scenarios.

Fig. 6. Visualization of the dense map created during the experimental deployment as a union of local maps used for precise navigation. The topological map is marked by cyan color. Parts of the environment and the wheeled robot are shown in the included photos, where an arrow from the robot's photo points to the robot's location.

The robot autonomously explores all reachable parts of the environment during the deployment while avoiding untraversable areas such as tables, chairs, and stairs, which sufficiently validate the local mapping. The Greedy strategy has been employed during the experiment, and the mission has been considered finished when the set of the possible targets W_t becomes empty.

B. Empirical Evaluation of Multi-robot Exploration

The proposed topological mapping has been validated using S.T.D.R. Simulator with three scenarios denoted S1, S2, and S3 based on the virtual cave circuit environments of the DARPA SubT, see Fig. 7. In all scenarios, five robots have been deployed, each with omnidirectional LiDAR with the range limited to $10\,\mathrm{m}$ providing laser scans at the height of $0.5\,\mathrm{m}$ above the terrain with the frequency of $10\,\mathrm{Hz}$. 6 DOF pose of the robot within the frame of the simulated environment is utilized for localization. In all the scenarios,

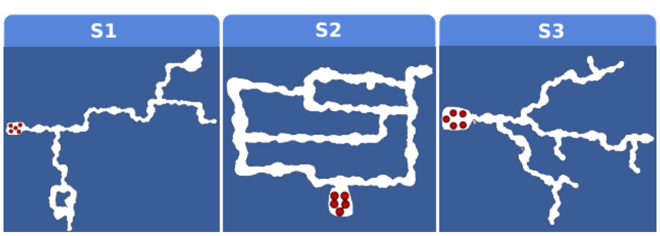

Fig. 7. Testing environments S1 – S3 used during the empirical evaluation. Red marks indicate the start locations of the robots.

ten simulation trials have been performed for each of the four coordination methods. Thus, overall, 120 trials have been performed for three scenarios.

The baseline methods **Greedy**$^{\mathbf{b}}$ and **MinPos**$^{\mathbf{b}}$ used a local mapping without limiting the range of the accumulated 3D point cloud and thus used a global map that is directly shared between the robots. Both **Greedy**$^{\mathbf{b}}$ and **MinPos**$^{\mathbf{b}}$ methods follow [3] and [19], however instead of frontiers, the waypoints are determined using the expected information gain. The methods with the proposed topological map sharing and adapted coordination strategies are denoted **Greedy** and **MinPos**. The performance is measured using p_{max} defined in (1). The achieved results are depicted in Table. III.

TABLE III

p_{max} [m] INDICATOR FOR EMPIRICAL EVALUATION

Setup	**Greedy**$^{\mathbf{b}}$	**Greedy**	**MinPos**$^{\mathbf{b}}$	**MinPos**
S1	412.0	389.8	311.6	280.5
S2	115.3	133.3	103.2	109.4
S3	192.2	194.7	171.2	212.0

The results indicate that the proposed approach is viable, and in S1, faster exploration is achieved. The **MinPos**$^{\mathbf{b}}$ outperforms all strategies in S2 and S3. However, the important characteristic of the topological mapping is in the reduction of the needed communication bandwidth that is reported in Table IV. The proposed topological mapping has significantly lowered demands on communication than sharing dense maps, but also less demands than [5]. Based on results in Table IV, it can be seen that only the proposed method fulfills the constrained bandwidth by broadcasting only $100\,\mathrm{B/s}$. Examples of robot paths obtained during the empirical evaluation are depicted in Fig. 8 and Fig. 9.

TABLE IV

COMMUNICATION REQUIREMENTS

	Packet size [B]	**Bandwidth** [kb/s]	**Robots** [-]
Proposed topological mapping	12	2	5
Reference map sharing	165220	6454	5
Compressed maps in [5]	56	33	3

Bandwidth calculated for the case all robots are mutually connected.

VI. CONCLUSION

In this paper, we present a novel mapping technique suitable for decentralized multi-robot exploration under low-bandwidth communication. The mapping is based on dense local mapping and sparse topological mapping based on exchanging the positions of the robots. The feasibility of both parts has been examined. The local mapping can also be used in the global map setup with dense map sharing. However, the topological mapping is significantly less communication demanding while the exploration is prolonged only about few percentage points. Thus, we found the proposed approach viable for scenarios where only very limited communication is available. Note that the low-bandwidth requirements of the proposed mapping are of great advantage for deployments with a huge number of robots.

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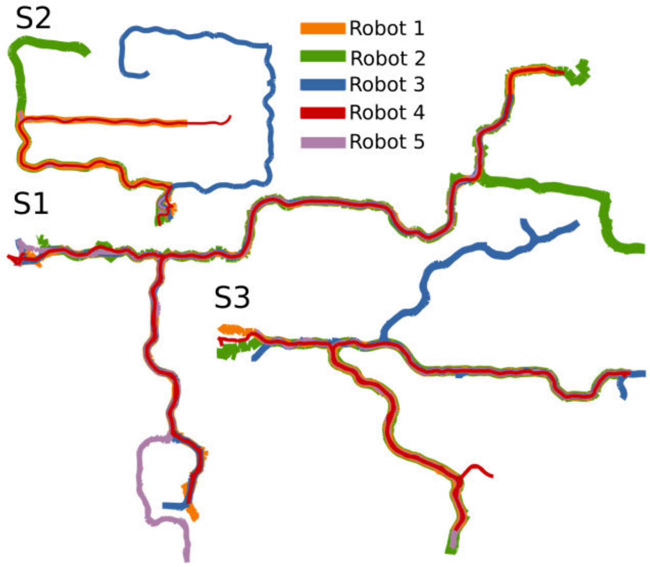

Fig. 8. Example of the robot paths in multi-robot exploration coordinated by method **Greedy**.

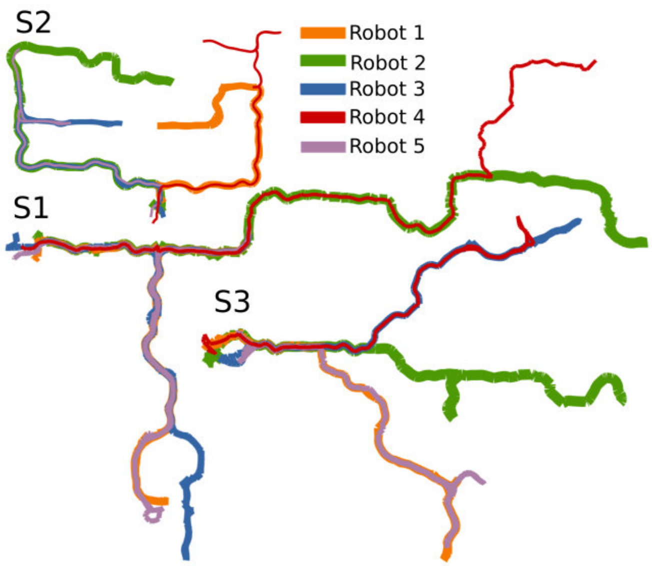

Fig. 9. Example of the robot paths in multi-robot exploration coordinated by method **MinPos**. Notice lower path overlap between the robots than in examples shown in Fig. 8.

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