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Multi-Goal Path Planning Using Self-Organizing Map with Navigation Functions

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Abstract. This paper presents a combination of Self-Organizing Map (SOM) approach and navigation functions in the Traveling Salesman Problem with segment goals where paths between goals have to respect obstacles. Hence, the problem is called multi-goal path planning. The problem arises from the inspection planning, where a path from which all points of the given polygonal environment have to be "seen". The proposed approach demonstrates applicability of SOM principles in such problems in which SOM has not yet been applied.

1 Introduction

The multi-goal path planning is a problem of finding a collision-free path for visiting a set of goals in a robot workspace [1]. This problem can be formulated as the Traveling Salesman Problem (TSP) in which a sequence of goals visits requested to be found such that the total length of the path is minimal. Self-organizing map (SOM) has been already used in the TSP and several approaches have been proposed, for examples surveys in [2, 3]. However, these approaches are focused on the Euclidean variant of the problem in which distances between neurons' weights and goals are efficiently computed as the Euclidean distances between two points. In the multi-goal path planning, obstacles have to be taken into account and geodesic distances have to be used otherwise poor solutions are found. Although only (to the best of our knowledge) approaches [4, 5] deal with obstacles, the approaches demonstrate applicability of SOM in the multi-goal path planning. The first one uses supporting graph and the second approach utilizes approximate shortest path between neuron's weights and goals presented to the network during the adaptation.

From the performance point of view, it is known that combinatorial approaches from the operational research domain provide better solutions of the TSP in less computation time than SOM based approaches [2]. The gap between heuristics and SOM is even wider in problems with obstacles as the path among obstacles has to be determined, which is obviously more computationally demanding. For TSP solvers working with a graph, all shortest path between goals can be precomputed and stored in a distance matrix. In the case of a polygonal representation of the problem, the paths can be found by Dijkstra's algorithm on a complete visibility graph [6]. Even though SOM can be used on such graph, its performance is worse than heuristic approaches.

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In this paper, we consider more general problem in which goals are sets of points rather than single points. This problem arises in the inspection planning in a polygonal map $\mathcal{W}$ where one can ask to find a path such that all points of $\mathcal{W}$ will be seen from at least one point of the path [7], e.g. finding an inspection path in a search and rescue mission [8]. Dividing the free space of $\mathcal{W}$ into a set of convex cells, the path can be found as a solution of the TSP where goals are diagonals of the map division. A diagonal is a line segment connecting two vertices of $\mathcal{W}$ such that it connects two nonadjacent vertices and it is contained in $\mathcal{W}$. An example of such segments is depicted in Figure 2a.

A variant of the TSP with goals represented as sets can be found as the TSP with neighborhoods (TSPN) in literature [9]. The goal is not a single point but it is a set of (possibly infinite) points, which means that the distance matrix cannot be easily precomputed, because too many eventualities are possible. In general, the TSPN is APX-hard and cannot be approximated within a factor 2, unless P=NP [10]. The difficulty of the TSPN is that the problem is not only to find a sequence of goals' visits, but also to select the most appropriate point of the set representing the goal. To address this difficulty, we propose a combination of the SOM based TSP solver with navigation functions used for determining the path from a point (neuron's weights) to the goal. The proposed approach demonstrates how SOM can be combined with motion planning techniques used in robotics. Moreover, the approach also shows that SOM based TSP solvers can be easily extended for more general problems, where they provide additional features over combinatorial heuristics.

The paper is organized as follows. The proposed approach is based on well known SOM adaptation schema, and therefore it is only briefly described in the next section. The navigation function is introduced in Section 3 as well as harmonic potential function used. The main contribution of the paper, the adaptation procedure for segment goals, is presented in Section 4. Finally, concluding remarks are presented in Section 5.

2 Self-Organizing Map for Routing Problems with Point Goals

Self-organizing Map (SOM) can be considered as a two-layered competitive learning network that has two dimensional input vectors and an array of output units. An input vector i represents coordinates (g_{i1}, g_{i2}) of the goal g_i and neurons' weights ν_{j1}, ν_{j2} can be interpreted as coordinates of the node ν_j in a plane. The output units are organized into a unidimensional structure and the connected nodes form a ring, which evolves in the problem domain according to the self-organizing adaptation procedure.

Although many variants of SOM algorithms for the TSP have been proposed, we consider the algorithm [11] as the main adaptation schema in this paper. A distance of a node to the presented goal is used for selection of the winner node, which is then (together with its neighbouring nodes) updated in order to be closer to the presented goal according to the neighbouring function f, i.e.

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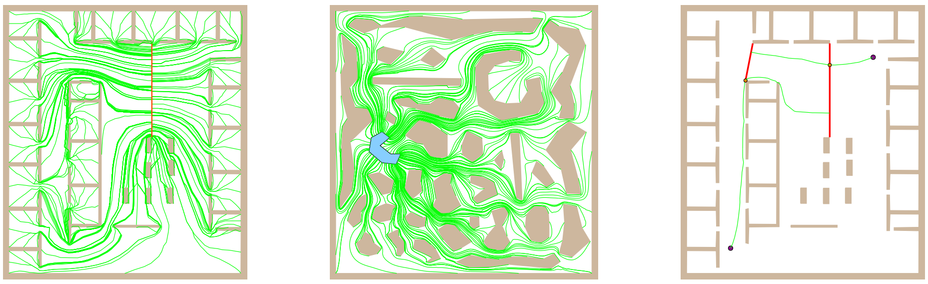
the new node position is $\nu'_j = \nu_j + \mu f(G,l)(g_i - \nu_j)$, where μ is the learning rate. The neighbouring function used is $f(G,l) = \exp(-l^2/G^2)$ for $l < 0.2m$ and $f(G,l) = 0$ otherwise, where G is called the gain parameter, l is the distance (in the number of nodes) of a node from the winner measured along the ring, and m is the number of nodes in the ring that is set to $m = 2.5n$, where n is the number of goals. G is decreased after each complete presentation of the goals to the network according to the gain decreasing rate α, i.e. $G = G(1-\alpha)$. The initial value of G is set using the formula $G_0 = 0.06 + 12.41n$ and learning and decreasing rates are set to $\mu = 0.6$ and $\alpha = 0.1$. After the complete presentation of all goals to the network, each goal has distinct winner node (due to the inhibition mechanism) and the final tour over all goals can be found by traversing the ring.

3 Navigation Function - Artificial Potential Field

The SOM algorithm relies on determination of the winner node using a node-goal distance. The winner is then moved towards the goal along the path to the goal. The problem of finding the node–goal path in $\mathcal{W}$ can be considered as a path (or motion) planning problem, thus any planning algorithm can be used. Here, we use a navigation function as a planning algorithm. The navigation function is basically a function providing a direction to the goal for a robot at any point in $\mathcal{W}$ and the goal will be reached if the robot follows the direction [12].



(a) (b) (c)

Fig. 1: Paths provided by navigation functions. (**a**) paths to a segment goal; (**b**) paths to a polygonal goal; (**c**) connection of segment goals.

The Artificial Potential Field (APF) technique [13] can be used as a navigation function. A potential function is a differentiable real-valued function f from $\mathbb{R}^m \to \mathbb{R}$ and its gradient $\nabla f(q)$ points in the direction of increasing f. A harmonic (potential) function satisfying Laplace's equation $\nabla^2 f(q) = 0$ has to be used as a navigation function to avoid local extrema [14]. A solution of the equation can be found numerically by the finite difference method or the finite element method (FEM). Particularly FEM is useful for polygonal environments, because finite elements can be represented by a triangular mesh. A combination of the Dirichlet boundary condition specifying f at the goal boundary and the

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Neumann condition specifying ∇f at obstacles is preferred [15].

The advantage of the APF (particularly FEM) is an ability to find a navigation function for a goal with an arbitrary polygonal shape, only boundary conditions need to be specified, see Figures 1a, 1b. Thus, a path to a goal with a general shape can be found as a sequence of points with desired smoothness computed from the sampling of the gradient in a particular point to the goal.

4 Self-organizing Adaptation with Segment Goals

Although the TSP with segment goals is similar to the point goals, the graph based algorithms for the TSP cannot be directly used. The main issue is that a destination point at the segment depends on the position at the previous segment in the tour. If the position of a vertex (representing the segment in the graph) in the tour is changed, the path can be completely different, which is a significant difference to the point goals where changes are only local.

The issue is not the case of the SOM adaptation, because the tour can be represented by the ring of nodes itself. The winner node is selected according to its distance to the goal, which is determined from the path provided by a navigation function. Such paths are also used to move nodes towards the goal. The adaptation procedure is terminated if each winner node is closer to the goal than the given distance δ. The final tour can be constructed from the winner nodes to the segment goals, because winner nodes can be negligibly close to the segment, e.g. $\delta \geq 10^{-5}$ m, and they represent particular points of each goal.

A tour (not only for negligibly close winners) can be constructed by the following procedure. Assume two winner nodes ν_a and ν_b associated to the segments s_a and s_b, see Figure 1c for an example. Due to the inhibition mechanism these two winners are distinct $\nu_a \neq \nu_b$. For each goal a navigation function is computed in advance. A path from the node ν to the segment goal s is determined by the navigation function $g_s(\nu)$ and the endpoint of the path at s can be denoted as $\text{end}(g_s(\nu))$. Then, the goals s_a and s_b can be connected as follows.

1. Determine endpoints of paths from ν_a and ν_b to the segments s_a and s_b; $\vartheta_a = \text{end}(g_{s_a}(\nu_a))$ and $\vartheta_b = \text{end}(g_{s_b}(\nu_b))$.
2. Two paths connecting s_a and s_b can be constructed:
   (a) a path from ϑ_a to s_b defined by $g_{s_b}(\vartheta_a)$ and a part of the segment s_b from the endpoints $\text{end}(g_{s_b}(\vartheta_a))$ and ϑ_b,
   (b) a path from ϑ_b to s_a defined by $g_{s_a}(\vartheta_b)$ and a part of the segment s_a from the endpoints $\text{end}(g_{s_a}(\vartheta_b))$ and ϑ_a.
3. The shorter path of these variants is selected and goals s_a and s_b are connected.

For a sequence of three goals s_a, s_b, s_c paths are firstly determined for each pair (s_a, s_b) and (s_b, s_c). After that, parts of the segments (collision free diagonals) defined by the particular endpoints are added to the length of the path from s_a to s_c over s_b. The final tour over all goals is found by the same schema. This

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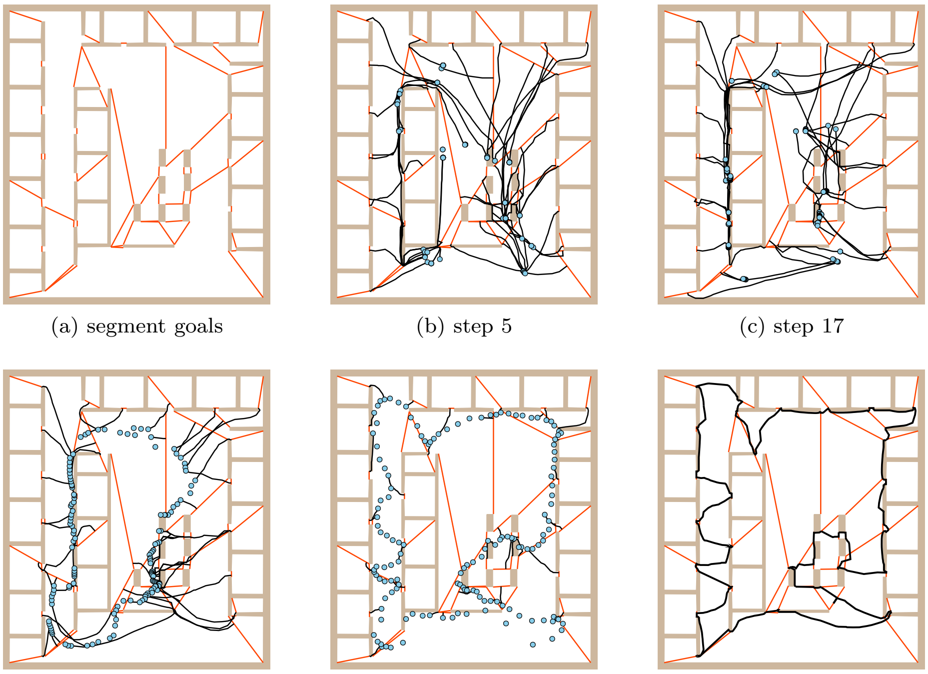
proposed procedure provides only approximate solution of the shortest path connecting given sequence of segment goals. The main aspect of the SOM approach is that a winner node becomes very close to the end point at the associated segment. Thus a point of the segment is selected during the adaptation, which is done naturally as nodes are moved towards the segments.

An example of nodes adaptations is depicted in Figure 2. In the first five sub-figures nodes are not connected, which is not necessary for adaptation, the black lines represent paths from the current winner nodes to the particular goals found by the navigation functions. The last sub-figure shows the final found path determined by the aforementioned procedure.



(a) segment goals (b) step 5 (c) step 17

(d) step 33 (e) step 47 (f) found solution

Fig. 2: An example of nodes evolution during adaptation.

5 Conclusion

The presented combination of the adaptation schema with navigation functions demonstrates advantages of SOM approach for more general variants of the TSP. At first, the approach is able to deal with obstacles, which has been noticed as a difficulty for SOM approaches. The other benefit is a straightforward extension for problems with general goals where SOM has not been used yet.

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Regarding the inspection planning, the advantage of the proposed approach is that a found path visiting diagonals of the convex partition guarantees inspection of the whole $\mathcal{W}$.

Future work in application of SOM principles in multi-goal path planning problems includes consideration of polygonal goals and additional motion planning techniques that will enable computation of paths among obstacles in high dimensional configuration space.

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