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Reaction-Diffusion based Computational Model for Autonomous Mobile Robot Exploration of Unknown Environments

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This paper introduces a computational model in which the main decision logic is based on principles arising from the dynamics of reaction-diffusion systems. The approach is an extension of our previous work where similar principles were used to develop a path planning algorithm. In this work, we select a mobile robot exploration task as a platform for exhibiting the core properties of the proposed computational framework. The functionalities represent particular building blocks that provide decision-logic capability of the exploration strategy. Beside a single mobile robot exploration, the proposed principles can also be generalized for multi-robot exploration, which is supported by the presented results.

Keywords: Reaction-diffusion equations, autowaves, exploration, mobile robots

1 INTRODUCTION

When surfing the rich tapestry of patterns arising when playing with non-linear systems, a persistent idea comes to mind: there is a huge amount of decision-logic taking place in the background as the pattern develops [3]. By means of a simple Reaction-Diffusion (RD) system, a wide range of behaviors can be reproduced: from simple excitations waves [19] to a more complex stable pattern formation through the so called Turing instability [23].

Beyond this, different instabilities or combinations of them [5, 24] have given rise to very intriguing static and dynamic structures. Moreover, the nuances found in the resulting concentration profiles reveal a mechanism that can be summarized in a single word: *self-organization*. A local transfer of information originates self-organize complex structures in a larger scale. Regarding this, it would be desirable to find out how to integrate this complex behavior into a conventional algorithm in order to take advantage of such implicit decision-logic as well as the nonlinear properties that RD systems exhibit.

Computationally, these concentration profiles are identified as simple geometric information, not surprisingly early works in this topic have already emphasized this point of view [11, 12, 14, 17, 19, 20, 26], subsequently extended by the motivation of introducing information into the system during the execution. This approach allows to codify such external information within the spatio-temporal dynamics, and constitutes the cornerstone of the present work. Herein it is used to solve the specific problems arising from practical deployment of mobile robots.

In short, the approach relies in a computer algorithm based on a RD-core whose different configurations encapsulate the necessary logic to perform suitable decisions during the execution. The basis of the computation consists of a set of independent blocks each one holding a different model configuration designed to exhibit a particular decision-logic capability. Upon this, an algorithm can be built putting together several of these blocks in sequence, being the output information of each stage the input of the next one. The communication with the rest of the algorithm takes place through the exchange of 2D geometric information as a set of I/O operations. It can be understood in terms of "black box", where some inputs deliver some outputs and where the main benefit is the ability to deal with complexity in a natural way, constituting an RD-based computational approach to decision-solving algorithms.

There is a vast amount of properties that can be extracted from the general dynamics of an RD system, though those described below provides a satisfactory framework for designing algorithms focused on robotic applications:

- Natural parallelism of the model evolution.
- Natural resistance to isolated damaged cells, interpreted as noise resistance.
- The possibility of choosing annihilation or non-annihilation of front-waves upon a collision. Therefore the information can be mixed or not at any time during the execution.

- Absence of pattern formation during the frontwave evolution that is spread out like an ordinary fluid, which leads to a background recognition inside the algorithm, after covering all available space.
- Change of relative stability in a bistable configuration leading to a switch of the direction of the fronts propagation.

Each one of these properties encapsulates a different decision-logic capability. For instance, the non-annihilation provides information about frontwave collisions, where the particular location of that collision remains a part of the system evolution. Although the inclusion of additional properties can extend the scope of applicability, we restrict to the aforementioned ones, which have been successfully applied to the robotic exploration.

The mobile robot exploration refers to an algorithm to create a map of an unknown environment, i.e., a problem, where a mobile robot is requested to autonomously cover all reachable regions by its sensory system. Although such an algorithm can be considered as tailored to accomplish a very specific mission, it provides a basis for decision-making principles throughout this work. Thus, it represents a special case of more general methods. Therefore herein is also used as a platform for introducing several capabilities of the proposed computational framework.

Beyond the basic feature of exploring an unknown area, a desired quality of the exploration algorithms relies upon the generation of a minimal-cost path by the exploring unit. This property can be specified by a distribution of the possible observation points, resulting in a set of points that minimizes the path length through the environment. In other words, this set of observation points guarantees an optimized route that a robot must follow to uncover with its sensory system the unknown environment, leading to an efficient exploration.

Therefore, measuring the performance of the exploration task is based on the strategy for determining the next goal. And consequently, the characterization of the exploration strategy is based on the optimizations used for the selection of such goals.

The basic exploration approach was introduced by Yamauchi in a seminal paper [28], along with the concept of frontier based exploration strategy. In Yamauchi's approach, the robot is navigated towards the closest location (the so-called frontier) in the limit between the known/unknown area. This approach provides a feasible solution of the exploration task and it forms a fundamental approach for more advanced exploration strategies [18], where additional improvements are in two streams. First stream focuses on determination of candidates from which next robot goal is selected. The second stream deals with the evaluation of navigation cost towards such goals. Thus, not only the distance cost is used, but also additional aspects can be considered like localization issues [22] or expected utility of the next goal [1], e.g., a combination of the distance cost with an expected information gain [8].

In this work, a new robot exploration algorithm fully based on RD processes is introduced and used for presenting a set of operations over the RD core. Although it is similar in many aspects to a regular frontier-grid-based approach, it provides a new way to generate goal candidates and to select the next goal. The approach is based on our previous work on computational autowaves applied to the path planning problem [25]. By considering the path planning method in the exploration task a new way to deal with multi-criteria cost function is provided.

The paper is organized as follows. A brief introduction to the underlying principles of the considered RD model and computational autowaves involved in the proposed approach are presented in Section 2 and Section 3. The proposed autowaves based exploration strategy is introduced in Section 4, evaluation of such strategy is presented in Section 5 together with a comparison with two standard exploration strategies. Besides, early results of the proposed computational principle applied to multi-robot exploration are presented. Concluding remarks are presented in Section 6.

2 BASIC AUTOWAVES BACKGROUND

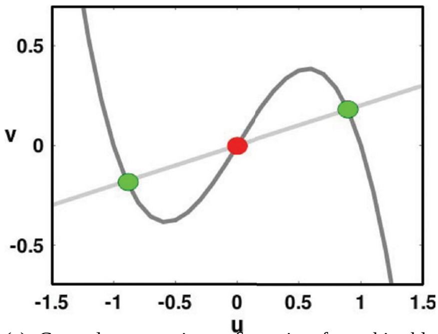
The fundamental principles of the RD dynamics considered with the basic methods for triggering frontwaves in particular configurations are already published in our previous work [25, 26] together with introduction of the nullcline configurations that characterize the model dynamics. Nonetheless, we shall provide an overview of such RD-based computation principles to increase the overall readability of this paper. Therefore, this section is a roundup of pillars for RD based computation model, followed by a more detailed description of specific properties used in the exploration task.

2.1 Reaction-Diffusion Computational Core Properties

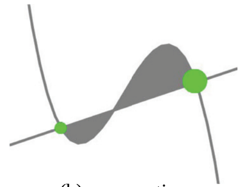
A reaction-diffusion system stands for an evolution of one or more substances, spatially distributed, by means of two processes. The diffusive process is responsible for spreading of the substances out in the space while in the reactive process the substances are transformed into each other. The considered mathematical model for describing the RD system is the FitzHugh-Nagumo (FHN) [7, 15] model, which adopts the basic form:

$$\begin{array}{rcl} \underline{u} & = & \varepsilon\left(u - u^3 - v + \phi\right) + D_u \triangle u \\ \underline{v} & = & (u - \alpha v + \beta) + D_v \triangle u \end{array} , \tag{1}$$

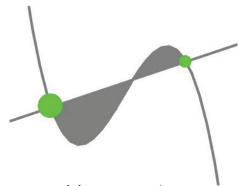
where α, β, ϵ, and ϕ are parameters of the model.



(a) General symmetric configuration for a bistable system. Both stable states are equally stable.



(b) propagation



(c) contraction

FIGURE 1
Nullcline configurations, the green discs represent stable steady states (SS) and the red disc represents the unstable state.

The reactive process has a local character and it is responsible for the dynamics of the model, which can be characterized by the associated nullclines (the geometric shape for which $\underline{u} = 0$ and $\underline{v} = 0$ in the absence of diffusion). Thus, in the so-called bistable configuration, the nullclines represent the shape depicted in Figure 1, where intersections of both curves define the *fixed points* of the system. These fixed points can be classified in stable states (in green, states towards which the concentration levels of the state variables (u, v) evolve naturally) and an unstable state (in red).

Asymmetric configurations that constitute a simple method for modulating the relative stability of both stable states (the larger area under the curve defines a bigger stability) are depicted in Figure 1(c) and Figure 1(b). Therefore, it is possible to differentiate between SS^+ and SS^- for each asymmetric configuration. Which represent a particular case for a more stable state and less stable state, respectively.

As the natural tendency of the system is evolving towards the concentration level of SS^+. A default configuration in SS^- causes the system to move to SS^+ when a small perturbation is introduced. The frontwaves driving this shift are well known for exhibiting different properties from standard waves, which is an attribute of the strong nonlinear character of the underlying model. Its behavior can be regulated by means of the nullcline configuration, it is possible to reproduce either frontwaves that annihilates after a collision, or conversely frontwaves that do not interfere after a collision, remaining static at the collision point. This fact contains a fundamental result from the information processing point of view: as long as collisions turns out in an active part of the model evolution, tracking of such collisions becomes unnecessary.

2.2 The Computational Model

The computation of the RD model itself takes place within a Cartesian grid in which the FHN model is discretized following the forward in time centered in space scheme (FTCS), i.e., the model evolution is performed within the integration grid. The discretization is bounded by Dirichlet conditions aimed to stop any wavefront propagation in the limits of the integration grid. Therefore, all computations rely on the dynamics of the spatially coupled cells endowed with the FHN model, where the short cell connectivity provided by the Laplacian term (coming from the associated diffusion process and which is responsible for the frontwaves propagation) is also the key allowing a simple hardware implementation.

The output of the model calculations consists of the concentration profiles for both variables: u and v. The employed process for the proposed decision logic is based on the dynamics of u; however, v provides a similar behavior.

Finally, the model allows us to consider additional constraints to the dynamics, which stands for an extra (point dependent) term aimed to represent the environmental information. Thus, the final expression to be discretized becomes a sum of the FHN model and the map grid containing information about the environment.

Note about the Computational Model – The proposed approach aims to use the evolution of the underlying RD model as much as possible for avoiding explicit computation, e.g., using computer graphics methods. The main interest of transferring as many operations as possible to the RD framework is because of possibility to use a hardware implementation for the underlying numerical models. This has been already discussed in [16] and some examples of practical realizations can be found in [2, 10, 27] providing a real time environment for the used operations. The particular techniques related to the exploration task are presented in Section 4.

3 PROPERTIES OF THE RD BASED COMPUTATIONAL MODEL

The underlying RD model provides the basic computational model exhibiting evolution of state variables u and v, which are used to build a decision-making logic for a particular problem. On top of this, we can exploit the model by considering its specific parameters and its behaviour after introducing additional information into the computational grid model. The following sections are dedicated to the particular properties that are used in the RD based computational framework for mobile robot exploration.

3.1 RD based Path Planning with Binary Forcing

The path planning problem is the problem of finding a shortest collision free path starting from some initial location and ending at the desired location, while a robot navigated along the path will avoid a collision with obstacles. A path planning algorithm based on the discussed RD computational principles has been introduced in [25] and the key point of the method is a consideration of the obstacles in a form of grid map used for forcing the RD model. The concept of the RD based planning is visualized in Figure 2.

The planning algorithm consists of two phases: *propagation* and *contraction*. The dynamics of the *propagation* phase is ruled by the nullcline configuration depicted in Figure 1(b), whilst the *contraction* phase is governed by the configuration shown in Figure 1(c). The algorithm works as follows. First, the system is initially set in $SS-$ and a small perturbation is introduced at the robot location, which triggers a frontwave to move the system towards $SS+$, see the upper-left picture in Figure 2. When the frontwave reaches the goal area (i.e., changing the concentration values to $SS+$), which can be

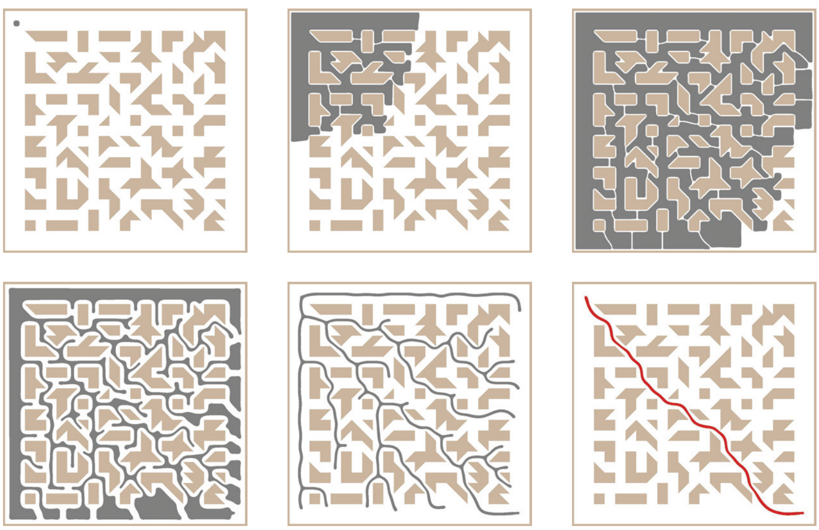


FIGURE 2
An example of the system evolution in the path planning problem; *upper*: propagation phase; *bottom*: contraction phase. The free space is in white and obstacles are in brown; values of the u state variable are in gray; the final solution is shown in red.

easily detected to constitute the termination condition for the propagation, the *contraction* phase is activated. During the contraction phase, the start and goal locations are simultaneously kept at the concentration levels of $SS-$ in opposition to the system tendency of evolving towards $SS+$.

The result of combining the changing of phase together with forcing the start and goal locations in the *contraction* phase is shown in the bottom row in Figure 2. The contraction of the domain $\left(SS^{-}\right)_{contract}$ over itself in benefit of the more stable domain $\left(SS^{+}\right)_{contract}$ leads to a final single path linking the start and goal locations, which corresponds to the requested shortest path.

3.2 Gradient-like Forcing: Modifying the Frontwave Velocity

In the previous section, a binary grid in the form of the grid map of the environment has been introduced as external information into the integration grid through the local modification of the nullclines of the particular point of interest (the obstacles). It means that at each point of the grid representing an obstacle the nullcline configuration is replaced by other that inhibits the wavefront propagation. This inhibition is suitable for introducing binary information into the integration grid and thus any propagating will stop at reaching any of these locations. However, it is also possible to introduce gradient-like information by means of an extra matrix (point dependent) F that is added to the model. Considering this term, the final form of the computational model is:

$$\begin{array}{rcl} \dot{u} & = & \varepsilon\left(u - u^3 - v + \phi\right) + F + D_u \triangle u \\ \dot{v} & = & (u - \alpha v + \beta) + D_v \triangle u \end{array} . \qquad (2)$$

Equation (2) is a common procedure for introducing different types of spatial or even temporal forcing in the model evolution. Based on this formulation, we can establish an effective range from zero to a specific value to gradually generate an increasing delay in the frontwave advance till the complete inhibition of the propagation. Therefore, environmental information can be introduced by means of the above described matrix, which slows down the frontwave propagation according to the predefined strength. An example of such a behavior is depicted in Figure 3, where a planar frontwave evolves according to a different values of F along the vertical axis: a maximum delay in the upper part of the front regarding the lower part where the value $F = 0$ means no forcing at all. Therefore, we can generate a profile that exhibits a gradient of velocities in the wavefront propagation along the vertical axis. This result is compatible with other RD properties used in the framework, like the annihilation upon a collision, and hence opens up a wide range of possibilities.

FIGURE 3
A profile of velocities in a wavefront propagation in 2D (300×700 grid size) under the influence of the F matrix, where the abscissa represents four states of the same evolving front at different times. The forcing matrix F is introduced along the vertical axis, with values in the range $\langle 0, -0.035 \rangle$.

4 EXPLORATION STRATEGY BASED ON AUTOWAVES

Although the proposed exploration strategy exhibits similar behaviour as regular frontier-based approaches, it is based on different underlying principles. That is why we split the description into a formal characterization of the algorithm, features of the autowaves applied in the exploration task, and proposed concept of utility consideration in the selection of the next goal for a mobile robot. The proposed approach follows standard exploration procedure in which the main loop can be defined by the following steps:

1. Integrate new sensor measurements into a map;
2. Determine goal candidates;
3. Select the next goal;
4. Navigate the robot towards the goal.

For the first step, we consider a common approach based on the occupancy grid [4]. However, the next three steps are based on principles of autowaves. In particular, instead of explicit Yamauchi's frontiers search, we use propagation of the frontwave to determine the possible goal candidates. The candidates correspond to frontiers; however, we can take advantages of the RD model dynamics and consider the determination of the candidates in a completely new way, see Section 4.1. The next goal is then selected by using the same principles used in the propagation of the frontwave. Finally, we employ the autowaves-based path planning [25] to determine the path to the goal along which the robot is then navigated. A detailed description of these three steps is presented in Section 4.1. Moreover, the new concept of dealing with utility of the goal candidates is proposed in Section 4.2.

The following assumptions need to be considered to expose the particularities of the exploring units and working environment in the proposed autowaves-based exploration approach:

- The map being explored consists of a series of objects and boundaries, all treated as binary information.
- Objects and boundaries are both represented by their edges.
- Each object or boundary within the sensor range is interpreted as an object–edge. Therefore, the exploring unit shapes the environment without any awareness about the correspondence between objects or boundaries.
- Since everything is treated as binary information, the discovered area is simply split into reachable and unreachable locations.

4.1 Determination of the Next Goal

The goal candidates are determined as a border of the currently known space of the environment. Here, the current map of the environment is used as a medium for exhibiting the RD model dynamics. The principle of the process is visualized in Figure 4. A frontwave (triggered from an excitable nullcline configuration) starts at the robot position and evolves as an ordinary

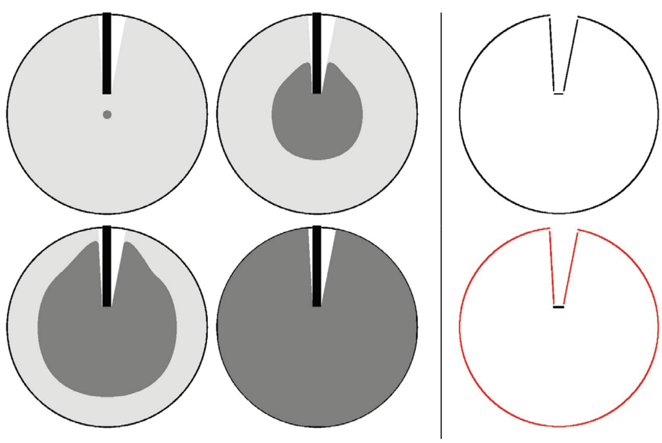


FIGURE 4
Calculation of the frontier using autowaves. Four left pictures represent a wavefront evolution. In particular, the light gray circular area corresponds to an omnidirectional sensor data (laser scanner), whilst the black rectangle in the top is an obstacle. The gray wavefront evolves from the robot position, covering all the discovered area till it reaches a static situation. The rightmost pictures represent the complete profile of the wavefront (top), and the final border reachable to the robot depicted in red (bottom).

fluid, covering all the available space (freespace in the integration grid) while adopting the shape of the space, and therefore, providing the boundary of the reachable area after reaching a static situation. Then, superimposing the obstacles (unreachable positions) results in the reachable border to the robot, as depicted in the red profile shown in Figure 4* .

The determined border corresponds to the frontier; however, the main difference is that we consider only reachable and unreachable parts of the space and all the logic of getting the contour is provided by the underlying dynamics of the RD system.

Once the border is determined, the next goal can be selected. Even though any path planning algorithm can be eventually used, we rather consider autowaves based approach [25]. The border does not distinguish the unknown cells from the obstacles. However, the frontwave propagation within the determined area given by the border takes advantage of the superimposing obstacles because autowaves preserved a distance (according to the wavelength of the frontwave) from the obstacles, see [25] for a more detailed discussion. An example of determining the next robot goal together with the determination of the path using the propagation and contraction phases is depicted in Figure 5. Another example of the goal determination is shown in Figure 9.

4.2 Utility Function

The aforementioned algorithm provides a basic approach for the exploration task and can be considered similar to the greedy frontier-based approach. The performance of the frontier-based exploration can be improved using the so-called utility function. It aims to consider an expected benefit of the frontiers to select the most suitable frontier in order to fulfill the mission objective, e.g., to collect the map of the whole environment as quickly as possible. Such approaches usually combine the distance cost with expectations about new areas, e.g., based on entropy [1]. Contrary to these approaches, we propose a different concept based on features of the frontwave propagation.

The dynamics of the underlying RD model allows to add an extra term representing additional information that is considered in the model evolution. This is numerically represented as a matrix (point dependent) term that can selectively introduce delays in the wavefront propagation, and therefore, it can prioritize some regions over others. This means that gradient-like information, rather than a binary map, can be used to represent the working environment. Consequently, the shortest-path technique for determination of the next goal can be switched into a more interesting lower-cost-path technique.

* Notice, the rectangular obstacle is drawn only for the visualization purpose as only its bottom edge is detected by the sensor system.

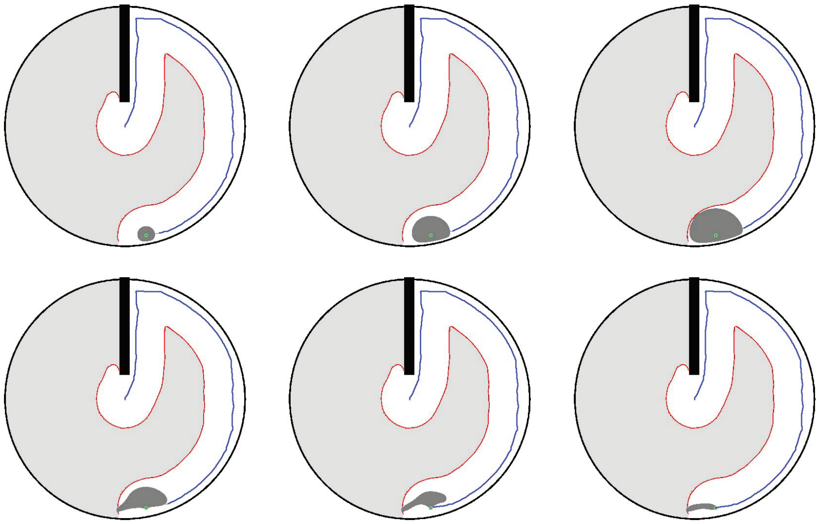


FIGURE 5
An example of determination of the next goal and the particular path towards the goal in a circular shaped environment. The already explored area is in white while the gray denotes not yet explored areas; the black color is the shape of the environment and the obstacles are superimposed only for a visualization purpose as they are not known during the exploration.

For example, we can establish a range $\langle -2, 0\rangle$ to the computational grid. Then, the range can discriminate states from the complete stop of the front-wave propagation (the value -2) over successively decreasing resistance till the effect vanishes (the value 0). In this sense, a very simple yet surprisingly useful utility function can be defined as follows:

1. Introduce an extra background (-0.35) within the explored area at every cell point, which in general slows down the wavefront evolution.
2. Remove the extra background from all cells representing objects as wells as their neighboring cells in the radius of 25 cells.
3. Change the value of the extra background from -0.35 to -0.25 for all cells at the border (frontiers-like cells) and their neighboring cells with the same radius 25 cells; thus, decreasing the resistance to the front-wave propagation.

Removing the extra background at the cells containing information about objects (obstacles) turns out into an increase of the wavefront propagation velocity around those places, and therefore, we get a basic boundary-follower

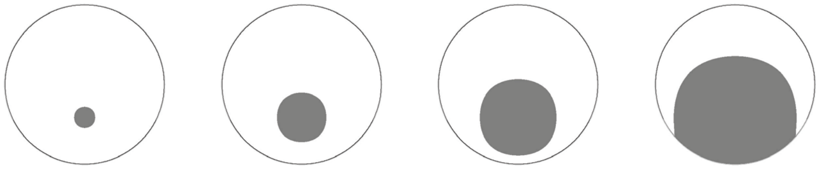


FIGURE 6
Demonstration of the utility introduction in the autowaves-based exploration approach. The circle represents a reachable area. The absence of obstacles allows the emerging frontwave to freely evolve from the robot position (slightly displaced downward relatively to the center of the circle) till the borders of the gray area, which in this particular case represents the boundary between explored and unexplored regions. Notice how the frontwave is not evolving to directly follow the circular shape since in the vicinity of the boundary, the forcing values have been deleted, which locally increases the speed of the frontwave.

exploring unit, which significantly reduces the tendency to perform multiple-visit to the same regions. A demonstration of such modified evolution of the frontwave is shown in Figure 6, where the growth of the frontwave in the vicinity of the wall is clearly faster than in any other direction.

5 RESULTS

The proposed building blocks of the RD core of the studied computational model have been verified in a series of exploration problems. The main intention of the presented results is to demonstrate feasibility of the proposed computational techniques and to provide an initial overview of the expected performance of the RD based exploration strategy. A natural performance metric for the exploration is the total time required to explore the environment, which can be estimated by the total traveled path. However, the real required time can be different according to the particular navigation strategy and low-level motion control. Thus, due to very high computational complexity of the underlying RD model implemented using a regular CPU the presented results focus on the travelled path† .

The section is divided into three subsections to show particular properties of the proposed RD based computational model. A demonstration of the alternative approach to the so-called utility based exploration is presented in Section 5.1, where the proposed concept of introducing preferences of areas to be explored by adding extra background is employed in a simple environment. In Section 5.2, the proposed computational principles of the selection

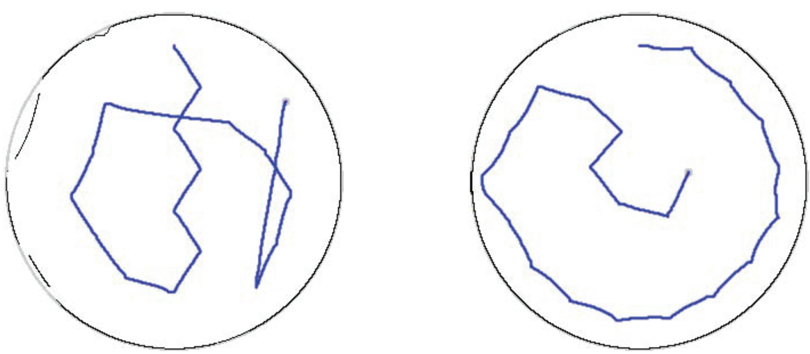
† The real-time factor is dedicated to our next work where a hardware parallelization will provide the required computational power.

of the next robot goal and the consecutive path planning (both based on the RD model) are employed in the exploration task by a single robot. In this case, the computational principles are integrated to the exploration framework introduced in [6]. The proposed computational model can be easily extended for multi-robot exploration, which is demonstrated in Section 5.3. A discussion of the achieved results is dedicated to Section 5.4.

5.1 Demonstration of Introducing Preferences of Areas to be Explored

The introduction of the extra background as the gradient like forcing allows to modify the velocity of the frontwave propagation; hence, it provides a mechanism to prefer exploration of specific parts of the environment, i.e., a kind of the utility function (see Section 4.2). This proposed concept has been verified in an exploration of a simple circular environment. In this simulation, the robot is equipped with an omnidirectional laser scanner with the sensing radius 5 m.

Two basic implementations of the utility function are visualized in Figure 7, which leads to two different behaviors. In the left case, the function is designed to prefer unexplored areas, this is achieved by decreasing the extra background along regions of the frontier that do not represent obstacles, therefore these regions become candidates for the next goal. As a consequence, the robot prefers to explore the open-space first. Conversely, in the right sub-figure, the background is especially decreased along detected obstacles, which increases the frontwave velocity along these regions. Therefore,



(a) open-space preference (b) wall-following preference

FIGURE 7
Visualization of the exploration behaviour for two different settings of the RD based computational model using extra background for modifying velocity of the frontwave propagation. The blue curve represents the traveled path of the robot to explore the whole environment.

once the robot discovers an obstacle, the next goal is naturally selected close to the already known obstacles, which provides a wall-following behaviour.

5.2 Performance of the RD based Exploration and Standard Approaches

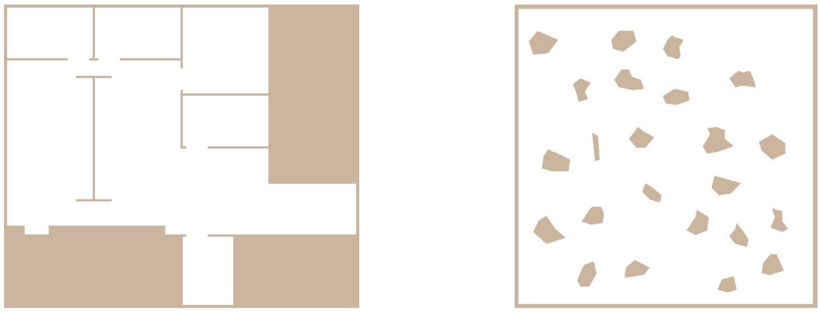
The proposed principles of the selection of the next robot goal and planning a path to the goal by means of propagation of the frontwave in the known reachable environment have been integrated into the exploration framework [6]. The integration provides an initial overview of the performance of RD based exploration strategy in comparison to standard approaches. The considered performance indicator is the path length of the robot to explore the whole environment.

The proposed RD based exploration strategy does not explicitly consider information gain, and therefore, it can be considered as a pure distance cost strategy. Therefore, we select the greedy exploration [28] and the TSP based approach [13] with the TSP distance cost for the comparison presented. However, a direct comparison of the approaches is not an easy task. It is because the proposed approach is based on a different computational model, where some properties directly accessible in regular frontier-based approaches are not straightforwardly and easily tunable. For example, a filtration of small frontier groups, which can significantly improve performance [9], and especially the planning horizon [6], i.e., a period of new goal determination. Thus, a direct comparison of the results would not be fair, and therefore, we include two variants of the greedy and TSP exploration strategies.

In the first variant, denoted by a subscript 1 (i.e., $greedy_1$ and TSP_1), the next goal is determined after the robot reaches its current goal (like in the RD based exploration), while in the second variant ($greedy_2$ and TSP_1), the next goal is determined after performing 7 navigational primitives, as it is suggested in [6]. In all these approaches, selected representative of the frontiers are considered as goal candidates using K-means algorithm, which decreases the computational burden of the TSP solution and also improves the exploration performance. A further description of this selection can be found in [13].

The RD based and greedy strategies are deterministic while the TSP based strategy is a stochastic method (due to the used approximate solution of the TSP). Despite that, we consider a single trial for each problem in this feasibility study and thus the presented results have only an indicative importance.

Two environments denoted as *autolab* and *potholes* are considered for the evaluation. The *autolab* environment represents an office like environment with dimensions 35×30 meters, and the *potholes* environment is a large open space (40×40 meters) with several obstacles, see Figure 8

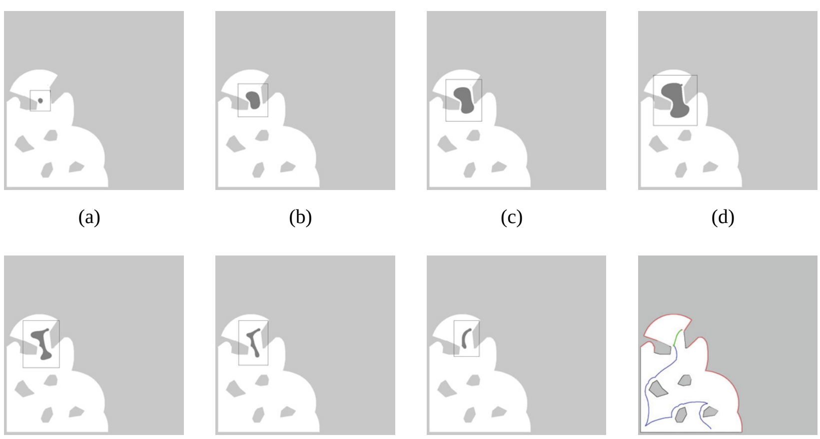


(a) *autolab* (b) *potholes*

FIGURE 8
Environments considered in the feasibility evaluation of the RD based exploration.

An example of the frontwave propagation during the selection of the next robot goal and the consecutive contraction phase of the used RD based path planning [25] is shown in Figure 9.

Indicative results are presented in Table 1, where the columns represent the length of the total traveled path (in meters) for the particular sensor range ρ. Notice, how a faster replanning period improves the exploration performance. On the other hand, for replanning after reaching the current goal,



(a) (b) (c) (d)

(e) (f) (g) (h)

FIGURE 9
Propagation of the frontwave towards goal candidates, selection of the next goal (**a**-**d**), an evolution of the RD model for the determination of the path to the selected goal (**e**-**g**) and the final found path is shown in green in the bottom rightmost picture; (**h**) the current travelled path is in blue.

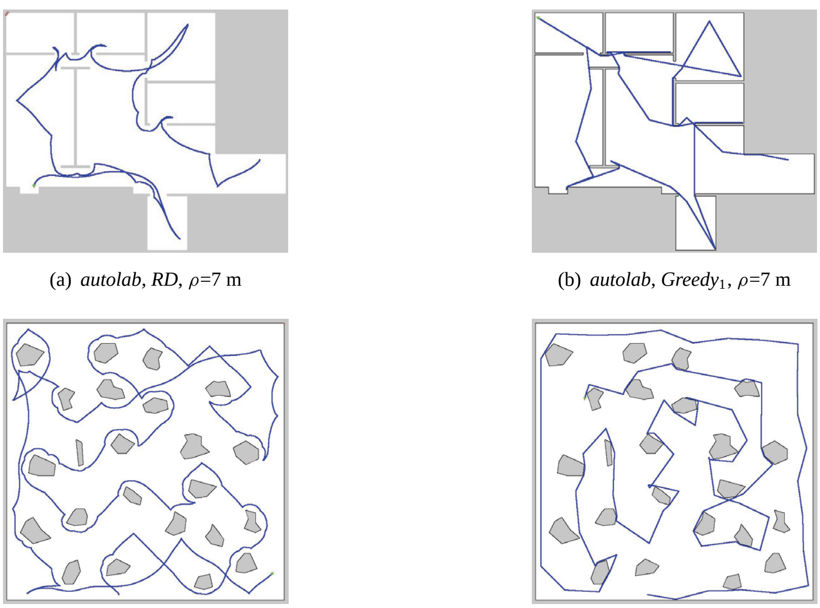
	autolab			*potholes*		
Algorithm	ρ =3 m	ρ =5 m	ρ =7 m	ρ =3 m	ρ =5 m	ρ =7 m
RD based	256.1	196.4	141.1	570.7	375.7	271.1
$Greedy_1$	262.5	232.5	182.9	545.8	398.8	320.4
TSP_1	263.0	222.8	174.9	543.9	342.5	283.0
$Greedy_2$	223.6	183.7	118.2	552.6	349.0	276.8
TSP_2	219.1	159.4	107.8	466.5	345.6	243.1

TABLE 1
Length of the Exploration Path

the RD based exploration provides competitive results to other approaches. Selected exploration paths for particular exploration strategies are depicted in Figure 10.

5.3 An Extension of the Proposed RD based Computational Model for Multi-Robot Exploration

The proposed computational model can be easily extended for exploration by a team of mobile robots. The extension can be considered as a



(a) *autolab*, *RD*, ρ=7 m

(b) *autolab*, $Greedy_1$, ρ=7 m

(c) *potholes*, *RD*, ρ=5 m

(d) *potholes*, TSP_1, ρ=5 m

FIGURE 10
Final exploration paths, the edges of the detected obstacles are in black while the unexplored areas ("inside the obstacles") are in gray.

semi-centralized approach, where the robots are considered independently while they shared a common map of the environment being explored. The proposed demonstration of the multi-robot exploration works as follows:

- Prior the determination of the next robot goal, each robot makes a local copy of the common map.
- The position of the other robots is introduced to the local map as solid objects; thus, robot considers the remaining robots as simple obstacles.
- A single step of the exploration algorithm is performed and new information is gathered from the environment.
- The local maps are merged into the common map (e.g., using the notion of occupancy grid and Bayes' rule like in [21]) and the process of determination of the robot goals is repeated.

The termination condition of the exploration is identical as in the single exploration algorithm, i.e., the exploration is terminated if all goal candidates (frontiers) have been covered. Examples of selected steps of the multi-robot exploration are visualized in Figure 11.

5.4 Discussion

The presented results indicate the proposed RD computational model based on the underlying dynamics of the highly non-linear system can be employed in the mobile robot exploration problem and it also seems the RD based exploration provides competitive exploration paths to the standard approaches.

Regarding the computational requirements of the proposed computational model based on the evolution of the RD model, it should be noted that it can take an advantage of the implicit parallel propagation; hence, it can be computed in parallel, and therefore, pruning all the possible solutions to the most adequate ones in parallel as well. This makes us optimistic about future implementation using massive parallel computation and a practical deployment of the method in real exploration problems.

Based on the presented results, we shall split our future work in two different streams. The first stream is to elaborate a detailed stability analysis of the diverse nullcline configurations used along this work to provide theoretical foundations for building blocks of the proposed RD based computational core. The second stream is to develop the exploration algorithm in order to perform quantitative comparisons with other state-of-the-art methods. This also includes study of the proposed concept of the utility towards exhibition of the benefits arising from a different computational model used for the decision-logic in the exploration missions.

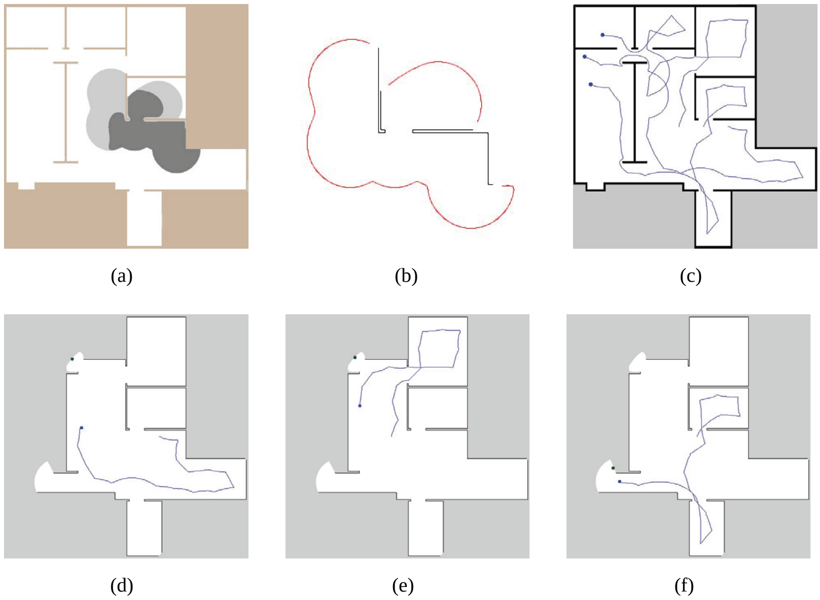


FIGURE 11
Examples of selected steps of the multi-robot exploration: (**a**) frontwave propagation during determination of the possible goal candidates (frontiers) superimposed to the map of the environment; (**b**) determined goal candidates (part of the map is enlarged for the sake of clarity); (**c**) final exploration paths for all robots; (**d**), (**e**), (**f**) same step of the exploration algorithm from the perspective of each robot (notice that all robots share the same map, corresponding to the whole discovered area in this particular step).

6 CONCLUSIONS

We present particular building blocks of a new computational model based on underlying principles of a reaction-diffusion model. The approach bears a huge potential for formalization due to its flexibility to be specialized and ease to incorporate new properties coming from the general dynamics of RD systems. In particular, herein, the approach has been successfully employed for solving the exploration task studied in mobile robotics. The approach constitutes a direct extension of our previous work where an autowaves-based path-planning was introduced, exhibiting interesting properties in comparison to standard approaches.

Although the presented results are preliminary, they indicate feasibility of the proposed RD based exploration and provide a premise for at least a competitive performance with the current state-of-the-art methods. The premise provides a ground for a further development towards a complete computational model for robotic tasks solely based on RD principles; thus, an approach where the decision logic is transfered to the RD core, which consistently provides ability of hardware implementation for real-time operations.

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