On the Dubins Traveling Salesman Problem with Neighborhoods

Petr Váňa, *Student Member, IEEE*, Jan Faigl, *Member, IEEE*

***Abstract*— In this paper, we address the problem of optimal path planning to visit a set of regions by Dubins vehicle, which is also known as the Dubins Traveling Salesman Problem with Neighborhoods (DTSPN). This problem can be tackled by a transformation to other variants of the TSP or evolutionary algorithms. We address the DTSPN as a problem to find Dubins path to visit a given sequence of regions and propose a simple iterative optimization procedure to find Dubins path visiting the regions. The proposed approach allows to efficiently solve the DTSPN and based on the presented comparison with existing approaches, the proposed algorithm provides solutions of competitive quality to the evolutionary techniques while it is significantly less computationally demanding.**

I. Introduction

Curvature-constrainted path planning for an unmanned aerial vehicle is a fundamental problem of surveillance missions where the vehicle is requested to visit a given set of static locations. The basic variant of this problem is known as the Dubins Traveling Salesman Problem (DTSP) [1] in which the problem is to find a shortest path with a bounded curvature (for Dubins vehicle [2]), such that the path visits a given set of single points in a plane.

A more general variant of this problem is a situation where particular waypoints can be selected from a set of possible locations. This is motivated by surveillance missions, where it is required to take a snapshot of each target location while each of such a snapshot can be acquired from a vicinity of the location [3]. Thus, it is not necessary to visit a particular location exactly, it is sufficient to visit just its proximity.

The variant of the TSP, where it is requested to find a tour that visits the given target regions instead of single point locations is called the Traveling Salesman Problem with Neighborhoods [4]. Based on this similarity, the problem addressed in this paper is called the *Dubins Traveling Salesman Problem with Neighborhoods* (DTSPN) [5].

Three classes of approaches to deal with the DTSP and DTSPN can be found in literature. The first class represents decoupled approaches in which a sequence of visits is determined independently on determination of the optimal Dubins path connecting the points [1], [6], [7]. The second class are transformation methods [8], [5], [9], [10] that sample possible configurations to visit the regions and the problem is transformed to a variant of the Asymmetric TSP, which is then solved using existing algorithms. Finally, the problem can be addressed by evolutionary approaches [11], [12].

The presented work has been supported by the Czech Science Foundation (GAČR) under research project No. GP13-18316P.

Authors are with the dept. of Computer Science, Czech Technical University in Prague, Technická 2, 166 27 Prague, Czech Republic {vanapet1|faiglj}@fel.cvut.cz

In this paper, we leverage on the recent reduction of the DTSP to convex optimization [13] and propose a new algorithm for the DTSPN. Its main idea follows the existing approaches where a sequence to visit the goals is determined prior determination of the Dubins paths; however, instead of explicit discretization of possible points of visits at each goal area, we rather employ an iterative local optimization technique to determine points of visits to the goals together with the particular headings of the vehicle at that points.

The proposed technique is based on the analysis of the properties of the optimal solution of the DTSPN. It allows to quickly find approximate solution of the Dubins planning problem for a given sequence of the region visits, which is called the *Dubins Touring Regions Problem* (DTRP) in the rest of this paper. Based on the presented comparison with existing approaches, the proposed algorithm provides first solutions for problems with hundreds of targets very quickly (in hundreds of milliseconds) using a conventional computer, while the quality of solutions is competitive to more computationally demanding evolutionary approaches.

The paper is organized as follows. An overview of the related work is presented in the next section. The addressed problem and considered assumptions are formally introduced in Section III together with the analysis of the optimal solution of the DTSPN and its transformation to the DTRP. The proposed iterative optimization algorithm is presented in Section IV and experimental results are discussed in Section V. The concluding remarks are in Section VI.

II. Related Work

Having Dubins vehicle [2] with a minimum turning radius ρ and a set of goal regions $\mathcal{R}$, the DTSPN is a problem to determine the shortest path that visits each goal region. The DTSPN is a generalization of the DTSP, where each goal is formed from a single point. As the DTSP is NP-hard [14], also its generalization in the DTSPN is NP-hard.

Dubins shows [2] that the optimal path connecting two points in a plane with a prescribed heading of the vehicle at the points is one of the six possible maneuvers (Dubins maneuvers), which are combinations of straight line segment and arc of exactly minimum turning radius. However, headings are not known a priori in our planning problem—DTSP(N) and we can imagine infinite possibilities of the heading at the points. Therefore, existing approaches for the Euclidean TSP (ETSP) cannot be directly used.

Three main types of approaches to deal with this difficulty can be found in literature. The first type are decoupling methods in which a sequence of visits is determined independently on the determination of the headings. The second type are transformation methods, where particular values of headings are sampled first and the problem is transformed to the Asymmetric TSP (ATSP) [12]. The third class of methods are evolutionary techniques, which are computationally demanding, but may provide high quality solutions.

Probably the simplest decoupled approach for the DTSP is the *Alternating Algorithm* (AA) proposed in [1]. It is based on the optimal solution of the ETSP to determine a sequence of visits to the goals. Then, headings are established in the way that even edges are connected by straight line segments and the odd edges correspond to the optimal Dubins maneuvers. The authors show that the length of the optimal solution of the DTSP can be bounded by $L_{TSP}\kappa\lceil n/2\rceil\pi\rho$, where L_{TSP} is the length of the optimal solution of the ETSP, n is the number of the goals, and $\kappa < 2.658$.

Based on the similar idea, authors of [6] proposed a receding horizon algorithm called the look-ahead (LA) approach to determine the heading at the next point in the sequence. The authors reported the LA algorithm provides superior solutions among AA and similar results are also reported in [15].

Optimal solution of the Dubins planning to visit a given sequence of waypoints that are at the distance longer than 4ρ is presented in [13]. The approach is based on convex optimization; however, the optimization needs to be solved several times because of possible alternation of the maneuvers directions. The authors bound the number of possible combinations to 2^{n-2} for n waypoints.

Transformation methods consider headings at the points are known and compute the length of Dubins paths between all pairs of the points. The distances and paths are used to create a complete graph that represents the original problem. Then, a solution is found in the graph by an ATSP solver. Authors of [9] proposed two variants of this approach: 1) with zero headings for all points; and 2) random values of the headings. Although approximation bounds are provided for both variants, the authors suggested to perform several trials of the randomized variant and select the best solution.

In the DTSPN with goal regions, not only headings have to be determined, but also the particular points of visits can be selected from an infinite set. The DTSPN can be addressed by the three aforementioned types of the methods. However, due to the regions instead of points, there are problems with disjoint and overlapping regions, for which particular algorithms may provide different performance.

Obermeyer et al. [3] propose a genetic algorithm to address the DTSPN with polygonal goals that may overlap and they report solutions up to 20 polygonal goals, but do not provide real computational requirements. Later on, the authors propose randomized sampling based resolution complete approach [8] that transforms the DTSPN into a variant of the Generalized TSP (GTSP) with mutually exclusive finite node sets. The GTSP is then transformed to the ATSP that is solved by the LKH algorithm [16]. In [8], authors report that the randomized sampling based algorithm is faster than the genetic algorithm [3] and solutions with 20 goals and 1500 random samples are found in several hundreds of seconds.

A similar approach has been proposed in [5], but Noon-Bean transformation [17] is used to transform the GTSP with overlapping node sets to the ATSP. The authors provide an analysis that the proposed method does not provide a worse solution than [8] while it is faster for problems with overlapping goal regions [10].

The DTSPN with overlapping disk goals is studied in [7]. First, a sequence of the visits is found as a solution of the ETSP using centers of the disks as the waypoints. Then, the number of waypoints for overlapping disks is decreased by the combination procedure [18] and the current path is further shortened by the proposed Alternating Iterative Algorithm, which provides alternative entry points of the disks to shorten the TSPN tour length. Finally, headings at the entry points are determined by the AA [1] for the DTSP.

An evolutionary approach may provide significantly better solutions of the DTSPN than the AA and sampling based approaches [11]. However, these techniques are computationally demanding and authors of the memetic algorithm [12] (for the DTSPN with the disk goals and relaxed terminal heading) report computational times 8.3 seconds and up to 45.5 seconds for problems with 10 and 17 goals, respectively.

III. Problem Statement

The addressed problem is motivated by UAV surveillance missions, where the vehicle dynamics is often modeled as the Dubins vehicle, which is going only forward at a constant speed and with the limited minimum turning radius ρ. The state of the vehicle q can be represented as the configuration $(x, y, \theta) \in SE(2)$, where $(x, y) \in \mathbb{R}^2$ is the vehicle position p in the plane and $\theta \in \mathbb{S}^1$ is the heading of the vehicle. The dynamics of the vehicle can be then described as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \frac{u}{\rho} \end{bmatrix}, \quad |u| \leq 1, \tag{1}$$

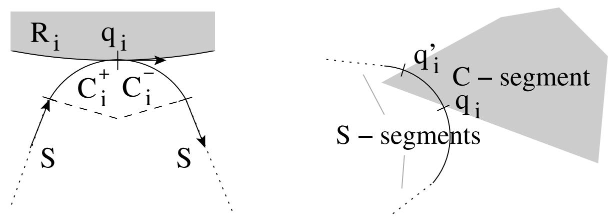
where v is the forward velocity of the vehicle and u is the bounded control input. For simplicity and without loss of generality, we consider $v = 1$ in the rest of this paper.

Dubins proved the optimal path connecting two configurations $g_1 \in SE(2)$ and $g_2 \in SE(2)$ consists of only straight line segments and segments with precisely the minimum turning radius ρ [2]. He also proved that such an optimal path can have at most 3 segments (where segments can have zero length) that can be categorized in two main types:

- CCC type: LRL, RLR;
- CSC type: LSL, LSR, RSL, RSR.

Now, we can formally introduce the problem being addressed in this paper as in [10]. Let $\mathcal{R} = \{R_1, \ldots R_n\}$ be a set of n regions $R_i \subset \mathbb{R}^2$ that are requested to be visited by Dubins vehicle and let $\Sigma = (\sigma_1, \ldots, \sigma_n)$ be an ordered permutation of $\{1, \ldots, n\}$. Define a projection from $SE(2)$ to $\mathbb{R}^2$, i.e., $\mathcal{P}(q) = (x, y)$, and let q_i be an element of $SE(2)$ whose projection lies in R_i.

The DTSPN stands to find the minimum length tour in which the Dubins vehicle visits each region R_i while



(a) C_i^+ and C_i^- have the same length and orientation

(b) C–segment does not satisfy property of the optimal solution

Fig. 1. C–segment of the CSC maneuver visiting a region R_i in the DTSPN with the D_4 constraint

satisfying the kinematic constraints of (1). This problem is an optimization problem over all possible permutations Σ and configurations q as follows:

Problem 3.1 (DTSPN):

$$\begin{aligned} \textit{minimize}_{\,\Sigma,q} \qquad & \mathcal{L}(q_{\sigma_n}, q_{\sigma_1}) + \sum_{i=1}^{n-1} \mathcal{L}(q_{\sigma_i}, q_{\sigma_{i+1}}) \\ \textit{subject to} \qquad & \mathcal{P}(q_i) \in R_i, \; i = 1, \ldots, n, \end{aligned}$$

where $\mathcal{L}(q_i, q_j)$ is the Dubins distance between q_i and q_j.

In this paper, we are focused on problems with regions R_i that are mutually exclusive at the distance longer than 4ρ. Formally, we can define the minimum distance constraint D_K on regions $\mathcal{R}$ such that for all $i, j \in \{1, 2, \ldots, n\}, i \neq j$ $\forall p_i \in R_i, \forall p_j \in R_j : \quad ||p_i - p_j|| > K\,\rho$.

A. On the Optimal Solution of the DTSPN

Here, we discuss properties of the optimal solution of the DTSPN for D_4 goal regions R_i. Our goal is to visit all the given regions and thus the solution of the DTSPN is a path with non-zero intersections with all the regions. Moreover, the optimal path connecting all the regions in the sequence Σ has to be composed of sequence of Dubins maneuvers between two consecutive configurations q_i and q_{i+1}. It is obvious that for every region R_i such a path has at least one configuration $q_i = (p_i, \theta_i)$ at the border of R_i.

Let $\mathcal{D}^* = (q_1, \ldots, q_n)$ be the optimal solution of the DTSPN with regions satisfying the D_4 constraint. We investigated properties of such a solution and the found insights are summarized as follows.

Lemma 3.1: For $\mathcal{D}^*$, all Dubins maneuvers between two consecutive configurations are always of the CSC type.

Proof: Euclidean distance between q_i and q_{i+1} is always longer than 4ρ, and therefore, it is not possible to construct the CCC maneuver [2]. ■

For the D_4 constraint, the maneuver between q_i and q_{i+1} is always of the CSC type and we can split the turn segment C_i corresponding to q_i into two parts: C_i^+ before reaching q_i; and C_i^- for the leaving part, see Fig. 1a. Notice, having configurations q_i (of the optimal solutions of the DTSPN) the optimal path is implied by Dubins maneuvers [2].

Lemma 3.2: For $\mathcal{D}^*$, all turn segments C_i, and corresponding q_i, it holds that both parts C_i^+ and C_i^- are equally long and they have the identical orientation.

Proof: We can fix the position p_i and let the orientation θ_i be free in $\mathcal{D}^*$. Then, the problem becomes the DTSP. Because we have the optimal solution of the DTSPN, we can also fix the permutation Σ and the problem becomes to determine the shortest path of bounded-curvature through a sequence of points. In [13], it is proven that an optimal solution of this problem under the D_4 constraint has the property under investigation. Thus, this property is inevitable also for all optimal solutions of the DTSPN with the D_4 constraint. ■

Lemma 3.3: For each region R_i in $\mathcal{D}^*$, there is only one configurations q_i for each R_i that is an intersection point with the optimal path such that q_i is the center of the corresponding C–segment C_i, or there is not a turn part of the optimal maneuver at q_i, i.e., $C_i^+ = C_i^- = 0$.

Proof: This can be shown by a contradiction. Let $\mathcal{D}^*$ passes a region R_i by a turn part at the configuration q_i. Assume, there are several intersections of the optimal path with R_i. Since q_i is a part of the optimal path, Lemma 3.2 holds and both parts C_i^+ and C_i^- are equally long. Now, consider for example a configuration q_i' in Fig. 1b, which is also a part of the optimal path. The optimal path consists of CSC maneuvers (Lemma 3.1) and thus q_i must be on the same C–segment as q_i. However, its corresponding C^+ and C^- parts are not equally long, which is in the contradiction with Lemma 3.2, unless there is not a turn segment or $C^+ = C^- = 0$. ■

Notice, Lemma 3.3 does not forbid more intersections of the optimal path with a particular region R_i as there can be infinite intersection points of the path with R_i. The results should be interpreted as a guideline, how to restrict possible candidates for the configurations q_i for which the optimal Dubins maneuver can be determined using [2].

B. Transformation of the DTSPN to the DTRP

Assume we have an optimal sequence Σ^* of visits to the regions $\mathcal{R}$ in the DTSPN. Then, the problem is to determine a configuration q_i for each region R_i such that the sequence of configurations prescribed by Σ^* is connected by the shortest path for Dubins vehicle with the minimum turning radius ρ. We call this problem as the *Dubins Touring Regions Problem* (DTRP) and it can be defined as follows.

Let Σ be a permutation of the regions $\Sigma = (\sigma_1, \ldots, \sigma_n)$ and $\Theta = (q_i, \ldots, q_n)$ be the corresponding configurations. Then, the problem is to find Θ that minimizes the total tour length $\mathcal{L}$:

$$\mathcal{L} = \mathcal{L}(q_n, q_1) + \sum_{i=1}^{n-1} \mathcal{L}(q_i, q_{i+1}). \tag{2}$$

A solution of the DTRP can be considered to find a solution of our original problem, the DTSPN. Since, we need a sequence of regions in the DTRP, we can imagine a generator of sequences for which we need a fast solution of the DTRP, which can be employed to prune not promising permutations of visits to the regions. In this paper, we focus on a quick solution of the DTRP, and therefore, we follow existing decoupled approaches and determine an initial sequence Σ as an optimal solution of the related ETSP.

The DTRP is addressed by a local iterative optimization procedure that is based on the properties of the optimal solution of the DTSPN with the D_4 constraint. The procedure performs an independent optimization of the heading θ_i and position p_i of each entry point $q_i = (p_i, \theta_i)$. Since p_i is always at the border of the region R_i, we can define a projection $\mathcal{A}$ from $\delta\mathcal{R}$ to $\langle 0, 1)$, i.e., $\mathcal{A}(q_i) = \alpha_i$ for $q_i \in \delta R_i$. This formal simplification of p_i as α_i allows to optimize the position as a single variable, and it is similarly introduced in [12].

IV. Proposed Iterative Algorithm for the DTRP

The proposed algorithm is based on the properties of the optimal solutions of the DTSPN with the D_4 constraint that follow the analysis [13]. The algorithm assumes a given candidate sequence Σ of visits to the regions $\mathcal{R}$ for which the DTSPN is transformed to the DTRP. The proposed DTRP algorithm starts with some initial configurations and for each region R_i it adjusts the configuration q_i to satisfy Lemma 3.1, Lemma 3.2, and Lemma 3.3. After an examination of all regions, the process is repeated until the path is not improving or a termination condition is not met, e.g., after a given number of iterations.

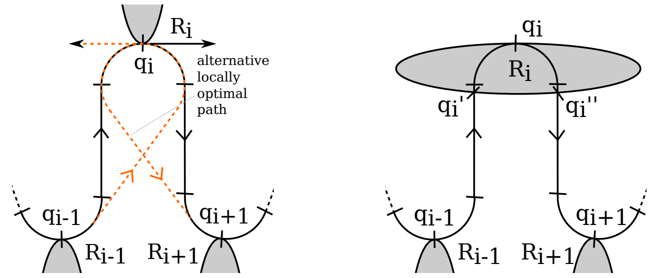
Algorithm 1: Local Iterative Optimization for the DTRP

```
Data: Input regions R, candidate sequence Σ
Result: Configurations (q_1, ..., q_n), q_i ∈ δR_i
initialization() // random assignment of q_i ∈ δR_i;
while global solution is improving do
    for every R_i ∈ R do
        θ_i := optimizeHeadingLocally(θ_i);
        α_i := optimizePositionLocally(α_i);
        q_i := checkLocalMinima(α_i, θ_i);
    end
end
```

The proposed algorithm is an iterative procedure in which particular candidate configurations q_i are locally adjusted by a consecutive optimization of the heading θ_i followed by an optimization of the waypoint position α_i. Then, a test for a local optimality of the new configuration q_i is performed to avoid a sub-optimal global solution. The algorithm is depicted in Algorithm 1 and it works as follows:

Let q_i be the current configuration under examination. Lemma 3.1 holds as q_{i-1} and q_{i+1} are at least in D_4 distance from q_i and thus the Dubins maneuvers between (q_{i-1}, q_i) and (q_i, q_{i+1}) are of the CSC type, see [13]. First, we evaluate a local optimality of the heading θ_i. If C_i^+ differs from C_i^- we adjust θ_i appropriately and Lemma 3.2 holds for q_i. We call this local optimization `optimizeHeadingLocally`.

Then, the `optimizePositionLocally` procedure evaluates if there is only one intersection of the path with R_i. If so, Lemma 3.3 is satisfied and we can switch to the next configuration q_{i+i}. Otherwise, we need to select a new point at the border of the region R_i. This is performed by a local hill climbing method, which adjusts α about a small increment $\delta\alpha$ until the total tour length $\mathcal{L}$ is not improving.



(a) Two locally optimal solutions (b) Local minimum of position α_i

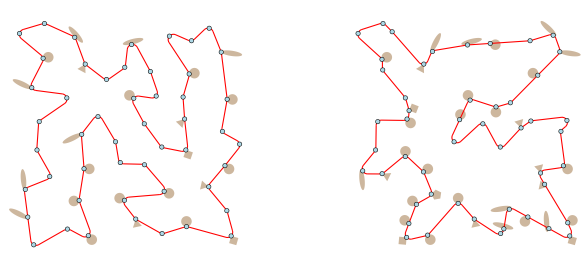
Fig. 2. Local extremes evaluated during the proposed local optimization

Finally, two additional tests are performed in the `checkLocalMinima` procedure to avoid local optima. First, an eventual exchange of the heading orientation θ_i about π is evaluated, which may shorten the tour length, e.g., see Fig. 2a. The second test is related to the position α_i, which is visualized in Fig. 2b. If this situation is detected, a position of q_i' or q_i'' (the one providing a shorter tour length) is selected for the value of α_i to escape the local minima.

Here, it is worth mentioning that the simple hill climbing optimization accompanied by `checkLocalMinima` does not necessarily find the optimal q_i regarding δR_i, for non-convex regions. For convex regions the proposed `checkLocalMinima` procedure covers most of the local minima cases, which are not discussed here due to the space limit. An empirical validation of the fast convergence is reported in Section V.

V. Results

The performance of the proposed algorithm has been evaluated in a series of scenarios. First, the algorithm has been evaluated for random instances of the DTSPN with convex regions and the D_4 constraint. In particular, the considered regions are points, disks with the radius ρ, ellipses with the semi-axis 2ρ and 0.5ρ, and random convex polygons with up to 6 vertices created from a disk with ρ radius. The problems with up to n=500 regions are randomly generated inside a bounding box with the side $6\sqrt{n}\rho$, which provides a relatively high density for the D_4 constraint. In addition, we relax the D_4 constraint and investigate the algorithm performance for closer and non-convex regions. An example of examined problems is depicted in Fig. 3.



(a) D_4 convex regions (b) D_1 convex regions

Fig. 3. Examples of the randomly generated instances of the DTSPN

The quality of the solutions provided by the proposed Local Iterative Optimization (LIO) is compared with two evolutionary algorithms [3], [12], which are able to provide high quality solutions of the general DTSPN. In addition, we compare LIO with the decoupled approach [7] based on the ETSPN, which is the most similar approach to the proposed LIO-based method. In both approaches (LIO and ETSPN-based), a sequence of visits is found as an optimal solution (using CONCORDE [19]) of the ETSP with centers of the regions. Therefore, the proposed approach is denoted as `ETSP+LIO` in the presented comparisons.

The evolutionary approach [3] is denoted as `Genetic` whereas [12] as `Memetic`. The ETSPN based method [7] is considered in two variants based on the procedure for determination of headings. The first variant denoted as `ETSPN+AA` utilizes the Alternating Algorithm (AA) [1] as in [7], while the second variant uses the proposed local optimization of the heading and it is denoted as `ETSPN+HoLIO`.

For each scenario (defined by n and the minimal allowed mutual distance between the goal regions) several random problem instances have been created with the minimum turning radius ρ=1. Based on the first results, we found out that the proposed method provides solutions with outstanding quality, and therefore, we evaluate the performance of the algorithms as the ratio of the final tour length to the solution found by the proposed ETSP+LIO algorithm.

All the algorithms have been implemented in C++ and run on a single core of the Intel Pentium E6300 CPU running at 2.8 GHz accompanied with 2 GB RAM. All the reported required computational times for the ETSPN-based and LIO methods include the required time to find the optimal solution of the underlying ETSP by CONCORDE [19].

Algorithms comparisons in the DTSPN with D_4 – The quality of found solutions regarding the dedicated computational time has been evaluated for problems with n=20 and n=40 goal regions. Due to the space limit, only the results for n=20 are shown in Fig. 4. Then, scalability of the algorithms for increasing number of the goal regions n has been evaluated for the computational time limited to 10 seconds, see results in Fig. 5.

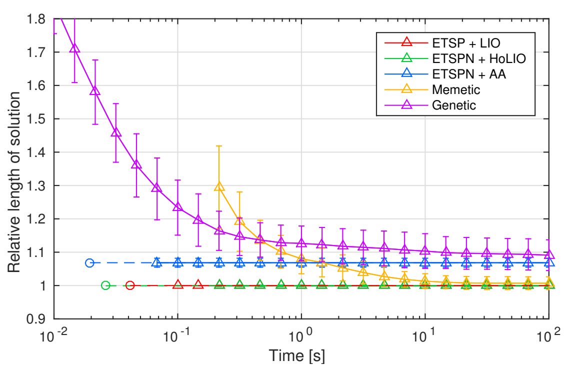


Fig. 4. The average ratio of the tour length (from 50 trials) according to the ETSP+LIO solution for the DTSPN with n=20 convex regions. Plots start from the time when the first solution is available.

Computational requirements of the proposed ETSP+LIO algorithm mostly depends on the optimal solution of the ETSP to determine the sequence of the visits to the goal regions. A detailed performance study of the LIO algorithm is depicted in Fig. 6, which shows quick convergence of the

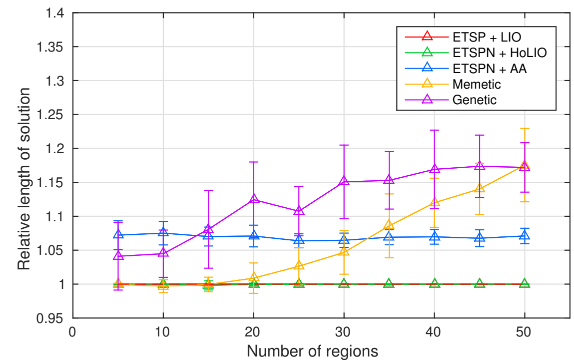
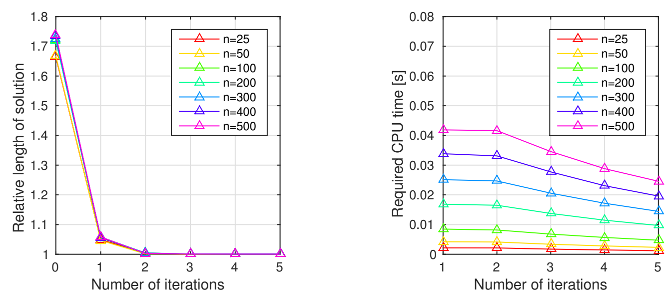


Fig. 5. The average ratio of the tour length (from 20 trials) according to the ETSP+LIO solution for problems with increasing number of regions. The Memetic and Genetic algorithms have been terminated after 10 seconds.

optimization in the first few iterations of the main while–loop (Line 2) of Algorithm 1 while it consumes a fraction of the required computational time.



(a) Average tour length (b) Average computational time

Fig. 6. Performance of ETSP+LIO in particular iteration of the main loop

Performance in DTSPN with relaxed D_4 constraint is depicted in Fig. 7 and examples of found solutions for the relaxed constraint are shown in Fig. 8.

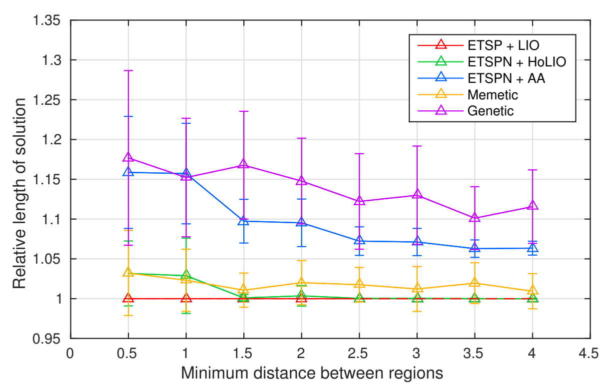
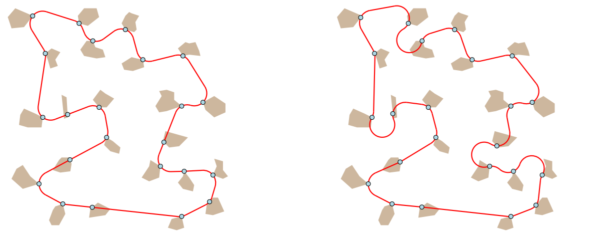


Fig. 7. Average ratio of the tour length (from 20 trials) to the proposed ETSP+LIO for increasing mutual distance of the goal regions. The Memetic and Genetic algorithms have been terminated after 10 seconds

A. Discussion

The presented results support feasibility of the proposed decomposition of the DTSPN into the DTRP and the idea of the local optimization based on the independent adjustments of the vehicle heading and entry point to the goal region. The proposed LIO algorithm provides solutions of competitive quality with significantly lower computational requirements



(a) ETSP+LIO, L = 70.73 m (b) ETSPN+HoLIO, L = 82.85 m

Fig. 8. Example of found solutions for the DTSPN with 23 regions that may be closer than D_4 and with ρ = 1 m

(about three orders of magnitude lower) than the evolutionary approaches.

Although LIO has been designed on top of the found properties of the optimal solution of the DTSPN with the D_4 constraint, the results for the relaxed constraint indicate suitability of the proposed approach also for general problems. The solutions quality provided by the ETSPN-based algorithm is competitive with LIO for D_4; however, it seems LIO provides better solutions for the problems with the relaxed constraint in comparison to other techniques. The results also support suitability of HoLIO as an alternative to the AA, which is faster but provides worse solutions, see Fig. 7.

The ETSPN-based method seems to be faster than the utilized hill climbing technique in LIO. Here, it is worth mentioning that we expect the performance can be improved by a more sophisticated implementation of the local optimization and parameters tuning. However, the presented results indicate that a selection of the entry points based on a fast computation of the Euclidean distance in the TSPN provides fast yet suitable estimation of the entry points to the regions. This estimation can be utilized in the proposed LIO method, which might reduce the number of the required computations of the Dubins path and thus decrease the computational burden.

Only a single sequence of visits to the regions is considered in the current decoupled approaches based on the solution of the ETSPN and ETSP, while the evolutionary methods are allowed to alternate the sequence, and therefore, they might find better solutions than the iterative algorithms. The proposed LIO algorithm for the DTRP converges very quickly, which makes it suitable to evaluate a set of candidate sequences to visit the requested goal regions. Therefore, the proposed idea to solve the DTSPN as the DTRP can be extended to consider generation of alternative sequences, which may further improve the solution quality.

VI. Conclusions

We propose to address the DTSPN by a transformation to the DTRP for which we propose a new algorithm based on the local iterative optimization (LIO) of the vehicle headings and entry points. The LIO method is based on properties of the optimal solutions of the DTSPN with the D_4 constraint. Although the proposed algorithm has been primarily developed for this restricted variant of the DTSPN, the presented results indicate a high performance of the algorithm in general problems where regions are allowed to be closer than 4ρ distance. The proposed LIO algorithm provides solutions of competitive quality to the evolutionary algorithms while it is about two or three orders of magnitude faster. A solution is found in less than 100 milliseconds using conventional computational resources, which makes it suitable for on-line deployment using an on-board computer of small UAVs.

References

[1] K. Savla, E. Frazzoli, and F. Bullo, "On the point-to-point and traveling salesperson problems for Dubins' vehicle," in *Proceedings of the American Control Conference*. IEEE, 2005, pp. 786–791.

[2] L. E. Dubins, "On curves of minimal length with a constraint on average curvature, and with prescribed initial and terminal positions and tangents," *American Journal of Mathematics*, pp. 497–516, 1957.

[3] K. J. Obermeyer, "Path planning for a uav performing reconnaissance of static ground targets in terrain," in *AIAA Guidance, Navigation, and Control Conference*, 2009, pp. 10–13.

[4] A. Dumitrescu and J. Mitchell, "Approximation algorithms for TSP with neighborhoods in the plane," *J. Algorithms*, vol. 48, no. 1, pp. 135–159, 2003.

[5] J. T. Isaacs, D. J. Klein, and J. P. Hespanha, "Algorithms for the Traveling Salesman Problem with Neighborhoods Involving a Dubins Vehicle," in *American Control Conference*, 2011, pp. 1704–1709.

[6] X. Ma and D. A. Castanon, "Receding horizon planning for Dubins traveling salesman problems," in *45th IEEE Conference on Decision and Control*, 2006, pp. 5453–5458.

[7] X. Yu and J. Hung, "Optimal path planning for an autonomous robot-trailer system," in *38th Annual Conference on IEEE Industrial Electronics Society (IECON)*, 2012, pp. 2762–2767.

[8] Karl Obermeyer, Paul Oberlin, and Swaroop Darbha, "Sampling-Based Roadmap Methods for a Visual Reconnaissance UAV*," in *AIAA Guidance, Navigation, and Control Conference*, 2010.

[9] J. Le Ny, E. Feron, and E. Frazzoli, "On the Dubins Traveling Salesman Problem," *IEEE Transactions on Automatic Control*, vol. 57, no. 1, pp. 265–270, 2012.

[10] J. T. Isaacs and J. P. Hespanha, "Dubins traveling salesman problem with neighborhoods: a graph-based approach," *Algorithms*, vol. 6, no. 1, pp. 84–99, 2013.

[11] D. Guimaraes Macharet, A. Alves Neto, V. Fiuza da Camara Neto, and M. Montenegro Campos, "An evolutionary approach for the Dubins' traveling salesman problem with neighborhoods," in *Proceedings of the 14th Annual Conference on Genetic and Evolutionary Computation*. ACM, 2012, pp. 377–384.

[12] X. Zhang, J. Chen, B. Xin, and Z. Peng, "A memetic algorithm for path planning of curvature-constrained uavs performing surveillance of multiple ground targets," *Chinese Journal of Aeronautics*, vol. 27, no. 3, pp. 622–633, 2014.

[13] X. Goaoc, H.-S. Kim, and S. Lazard, "Bounded-curvature shortest paths through a sequence of points using convex optimization," *SIAM Journal on Computing*, vol. 42, no. 2, pp. 662–684, 2013.

[14] J. Le Ny, E. Frazzoli, and E. Feron, "The curvature-constrained traveling salesman problem for high point densities," in *46th IEEE Conference on Decision and Control*, 2007, pp. 5985–5990.

[15] P. Isaiah and T. Shima, "Motion planning algorithms for the Dubins Travelling Salesperson Problem," *Automatica*, vol. 53, pp. 247–255, 2015.

[16] K. Helsgaun, "An Effective Implementation of the Lin-Kernighan Traveling Salesman Heuristic," *European Journal of Operational Research*, vol. 126, no. 1, pp. 106–130, 2000.

[17] C. Noon and J. Bean, "An Efficient Transformation of the Generalized Traveling Salesman Problem," Department of Industrial and Operations Engineering, University of Michigan, Ann Arbor, MI, USA, Tech. Rep. 91–26, 1991.

[18] L. He, J. Pan, and J. Xu, "Reducing data collection latency in wireless sensor networks with mobile elements," in *IEEE Conference on Computer Communications Workshops*, 2011, pp. 572–577.

[19] D. Applegate, R. Bixby, V. Chvátal, and W. Cook, "CONCORDE TSP Solver," 2003, [cited 28 Feb 2015]. [Online]. Available: http://www.tsp.gatech.edu/concorde.html