On Finding Time-efficient Trajectories for Fixed-wing Aircraft Using Dubins Paths with Multiple Radii

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ABSTRACT

Trajectory generation for fixed-wing aircraft can be based on Dubins vehicle model that constrains the vehicle to move forward with a limited turning radius and a constant speed. However, such a vehicle model cannot benefit from the physical capabilities of a real fixed-wing aircraft that can adjust its speed. Therefore, we propose to address the limitation of Dubins vehicle model by a generalized model that combines various turning radii, and thus allows increasing the cruise speed whenever possible. The proposed method provides faster trajectories in comparison to the trajectory generated by Dubins vehicle with a single turning radius and a constant cruise speed. The benefit of the proposed method is demonstrated on point-to-point trajectories, for which the parameters are inspired by Cessna 172 aircraft.

KEYWORDS

Dubins vehicle, Travel time estimation, Multiple radii, Time-optimal trajectory, Speed profile

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1 INTRODUCTION

In this paper, we study the problem of trajectory planning for fixed-wing aircraft, where the goal is to find time-efficient trajectories while the motion constraints of the vehicle are fulfilled. Most of the fixed-wing vehicles are limited by the minimum turning radius, and therefore, a model called Dubins vehicle (or Dubins car) [5, 9] is often used. The model represents a non-holonomic vehicle with a constant forward speed and a fixed minimum turning radius. The shortest path connecting two points with the prescribed leaving and arrival angles of Dubins vehicle (two configurations) can be computed efficiently by a closed-form expression [2].

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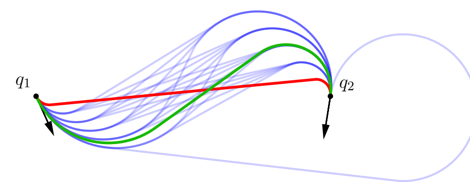


Figure 1: Example of generated Dubins trajectories for various combinations of the initial and final turning radii (blue). The fastest trajectory is in green, and the shortest one in red.

Various extensions of Dubins vehicle have been proposed in the literature. Reeds and Shepp [14] considered a bi-directional vehicle. Furthermore, the three-dimensional extension called the Dubins Airplane model [4] can be utilized if altitude changes are required. Time-optimal Dubins paths under steady wind field are studied in [1, 10, 15] and for the unsteady field in [11], while unknown wind disturbances are studied in [20].

We propose to extend Dubins vehicle model to utilize longer turning radii than the minimum radius because it enables us to generate faster trajectories; see example in Fig. 1. The concept of multiple radii have been already utilized for finding time-optimal [19], energy-optimal trajectories [16], and safe emergency landing trajectories [17]. However, the herein proposed approach respects the limited value of the forward acceleration, in contrast to the time-optimal trajectories [19] that contain discontinuities in the speed. The proposed heuristic approach utilizes Dubins path with two turn segments and the central straight segment, for which both turning radii are optimized to get the fastest trajectory possible. The travel time of the trajectory is computed based on *speed profile* that takes into account both speed and forward acceleration limits. Although an alternative approach can be based on parametric curves, such as Bézier curves [6], the presented approach exploits computationally efficient closed-form solution of Dubins path [5], which can be determined in microseconds [18].

The text is structured as follows. The time-optimal planning problem is introduced in the following section. The proposed method using multiple radii is described in Section 3, and computational results are presented in Section 4. The final remarks are in Section 5.

2 PROBLEM STATEMENT

The problem studied in this paper is to find the fastest (time-optimal) trajectory for a fixed-wing aircraft between two configurations. The aircraft is modeled as an extended version of Dubins vehicle [5] for which the speed is not constant and may be changed to shorten the travel time. The state of the vehicle q is represented by the configuration $(x, y, \theta) \in SE(2)$, where both positions $(x, y) \in \mathbb{R}^2$ and heading angle $\theta \in \mathbb{S}^1$ are given. The dynamics of the vehicle depends on the forward speed v and curvature of the trajectory κ.

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = v \begin{bmatrix} \cos\theta \\ \sin\theta \\ \kappa \end{bmatrix}. \quad (1)$$

The curvature κ is a scalar value, and its sign determines the left/right orientation of the turn in the plane.

$$\kappa = \frac{x'y'' - y'x''}{\left(x'^2 + y'^2\right)^{\frac{3}{2}}}. \quad (2)$$

The magnitude of κ is constrained by the minimum turning radius $R_{\min}$ of the vehicle

$$|\kappa| \leq \frac{1}{R_{\min}}. \quad (3)$$

The speed v and the magnitude of the forward acceleration a of the vehicle are limited by the minimum and maximum values.

$$v \in [v_{\min}, v_{\max}], \qquad a \in [a_{\min}, a_{\max}]. \quad (4)$$

Furthermore, the curvature restricts the maximum speed of the vehicle such that

$$v \leq \sqrt{\frac{g \tan \varphi_{\max}}{|\kappa|}}, \quad (5)$$

where $\varphi_{\max}$ is the maximum bank angle of the vehicle and g is the gravitational acceleration. The constraint (5) ensures that the bank angle can compensate centrifugal force, and the vehicle can fly in a steady configuration with zero sideslip.

The problem is to find the time-optimal trajectory between two configurations $q_1, q_2 \in SE(2)$ while all defined motion constraints are met. The problem is formulated as the optimization Problem 2.1 to minimize the travel time $T(\Gamma)$ necessary to execute the trajectory Γ.

Problem 2.1 Time-optimal Planning**.**

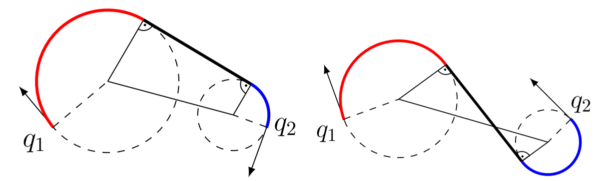
$$\begin{aligned} \min_{\Gamma} \quad & T(\Gamma) \\ \text{s. t.} \quad & \Gamma : [0, s_{max}] \to SE(2) \\ & \Gamma(0) = q_1, \quad \Gamma(s_{\max}) = q_2 \\ & \text{Equations (1–5) are fulfilled} \end{aligned}$$

3 PROPOSED METHOD

The proposed approach to find time-optimal trajectories is based on the extended Dubins vehicle model defined by (1–5). The radius of Dubins path is adjusted for each of the turn segments such that the overall travel time is minimized. The travel time is further denoted as the *Travel Time Estimation* (TTE).

3.1 Multi-radius Dubins Paths

An independent turning radius is assigned to each turning segment of the multi-radius Dubins path, which allows increased speed, and thus reduced TTE. The multi-radius Dubins path is based on the original Dubins path [5] with a fixed turning radius, and consists of three segments. There are two possible segment types: a curve segment C further specified as right or left denoted R/L; and straight segment S. Thus, two classes of paths CSC (LSL, RSR, LSR, RSL) and CCC (LRL, RLR) can be constructed, and gives six individual paths in the total. However, only CSC maneuvers are considered for the multi-radius extension because CCC maneuvers may occur only if initial and final configurations are very close [8]. An example of multi-radius CSC Dubins paths is shown in Fig. 2.



(a) RSR maneuver **(b) RSL maneuver**

Figure 2: Examples of multi-radius CSC Dubins paths.

3.2 Computation of the Travel Time Estimation (TTE)

Dubins path contains three segments with fixed curvatures, and thus a maximum speed can be computed separately for each segment according to (5). For the central straight segment, the speed is limited only by the maximum value for the specific aircraft $v_{\max}$.

There are three possible cases that can occur based on the length of the center straight segment. The first case occurs if the straight segment is long enough to accelerate to the maximum speed. In the second case, the maximum speed cannot be reached. In the last case, the vehicle is only able to accelerate/decelerate from the initial to the final speed. The cases are depicted in Fig. 3, where the segment colors correspond to Fig. 2.

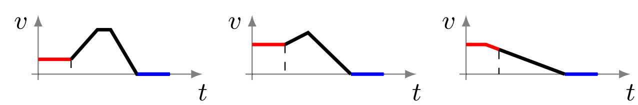


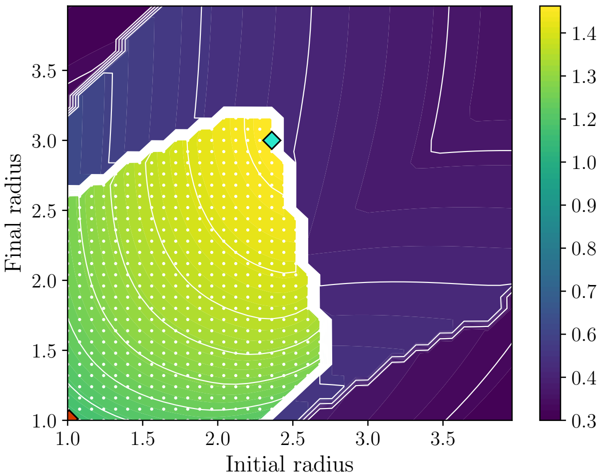
Figure 3: Speed profiles based on segment lengths.

4 COMPUTATIONAL RESULTS

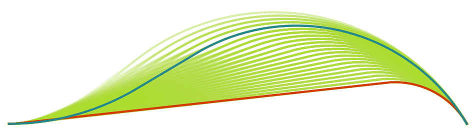
The proposed multi-radius Dubins path approach has been evaluated in two scenarios for the vehicle model parameters according to the Cessna 172 aircraft [3]. The minimal speed $v_{\min} = 30.0\,\text{m}\,\text{s}^{-1}$ has been selected greater than the stall speed to guarantee safe flight, and the maximum speed $v_{\max} = 67.0\,\text{m}\,\text{s}^{-1}$ is taken directly from the manual. The limits of turning radius are computed from $v_{\min}$ and $v_{\max}$ using the maximal bank angle $\varphi_{\max} = 60.0°$; the minimal turning radius $R_{\min} = 65.7\,\text{m}$ and the maximal radius $R_{\max} = 264.2\,\text{m}$.

In the first scenario, the influence of multiple turning radii to the TTE is studied in a simple case shown in Fig. 4. The improvement of the TTE is measured as the achieved speedup for the particular trajectory in comparison to Dubins path for the minimal turning radius. The achieved speedups are shown in Fig. 4a with an example of the generated trajectories in Fig. 4b, where the fastest trajectory is in the green, the shortest trajectory is in the red, and the particular values of the radii are highlighted by a-diamond shape in Fig. 4a. The discontinuities in Fig. 4a are caused by the maneuver type change when the turning radii become too large to construct the maneuver with the original type.

The achieved speedup of the TTE is further studied for a discrete set of turning radii. The problem is also solved using L-BFGS [13]



(a) Achieved speedup of the TTE of multi-radius Dubins trajectory in comparison to the trajectory with the single minimal turning radius.



(b) Paths for turning radii as the white dots in the upper figure.

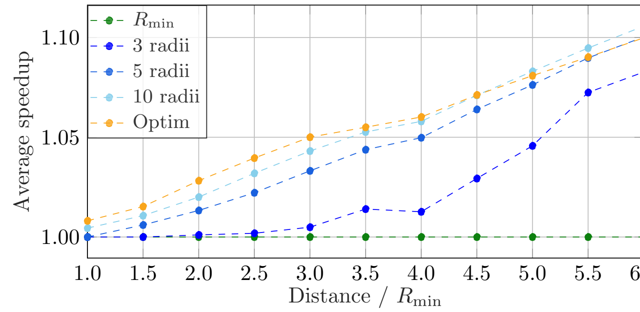
Figure 4: Influence of the initial and ending radii to the TTE for the endpoints $5.5\,R_{\mathbf{min}}$ far, initial heading angle $\theta_1 = 0°$, and arrival angle $\theta_2 = 300°$.

method from the optimization framework Optim [12]. For each set, 100 random instances have been generated and the TTE speedup and computational times are reported in Fig. 5 as average values.

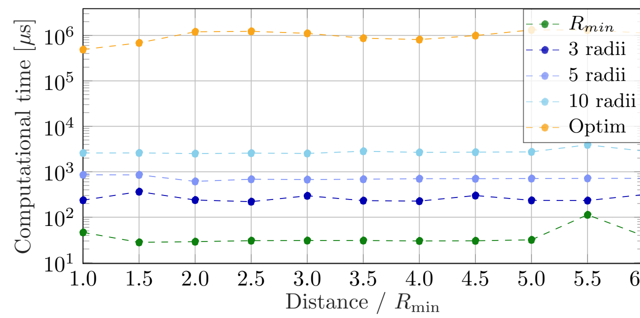
All the optimization methods have been implemented in Julia v1.2 and run using the Intel CPU i7-8550U running at up to 4.0 GHz. The average computational time of multi-radius Dubins path with the corresponding TTE takes 7.76 ms for a discrete set of radii, but using numeric optimization of the Optim framework takes about 888.00 ms due to large number (≈1000) of candidate trajectories.

5 CONCLUSION

The proposed multi-radius Dubins path provides a way how to exploit the variable speed of the vehicle and finds a heuristic solution of the time-optimal trajectory. The found trajectories are about 10 to 30 % faster for most of the evaluated instances in comparison to Dubins path with the minimum turning radius. In future work, we aim to utilize the proposed heuristic in combinatorial planning problems [7], such as the Dubins Traveling Salesman Problem.



(a) Average speedup of the TTE in comparison to Dubins path.



(b) Average required computational time.

Figure 5: Performance comparison of the multi-radius optimization methods to Dubins path with the radius $R_{\mathbf{min}}$.

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