Wireless Communication Infrastructure Building for Mobile Robot Search and Inspection Missions

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***Abstract*— In the paper, we address wireless communication infrastructure building by relay placement based on approaches utilized in wireless network sensors. The problem is motivated by search and inspection missions with mobile robots, where known sensing ranges may be exploited. We investigate the relay placement, establishing network connectivity to support robust flood-based communication routing. The proposed method decomposes the given area into Open space and Corridor space where specific deployment patterns allow for guaranteed k-connectivity, making the resulting network redundant while keeping channel utilization bounded. In particular, a hexagonal tesselation coverage pattern with 3-connectivity is investigated in Open space and a linear 4-connectivity pattern in Corridor space, respectively. The proposed approach is empirically evaluated in a realistic scenario, and based on the reported results, it is found superior compared to the existing stochastic randomized dual sampling schema.**

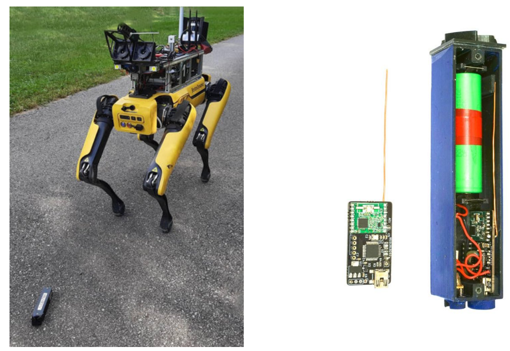

Fig. 1. A motivational real robotic system for the addressed wireless communication infrastructure building. The communication module depicted on the right is a low-power $868\,\mathrm{MHz}$ transceiver implementing flood protocol to facilitate the role of communication relay in considered robotic missions.

I. Introduction

In mobile robotics search, inspection, or even patrolling missions, wireless communication is necessary for timely information delivery and mission progress monitoring and control [1]. However, establishing a direct communication link in communication-denied environments might not be possible due to distance or obstacles. A standalone communication infrastructure can be built using communication relays to facilitate information exchange [2]. A mobile robot can be equipped with a set of communication relays to be deployed in the environment, such as depicted in Fig. 1, to create a connected network allowing mutual communication between the robots operating in the environment and enabling communication with the mission control [3].

As seen in the recent DARPA Subterranean Challenge (SubT) [4], contemporary methods rely on reactive rules [5] or human supervisor decision [6] to deploy radio relays ad-hoc. On the other hand, the relay placement can be considered similar to the placement strategies studied in *Wireless Sensor Network* (WSN) to deal with a set of localized sensors with limited transmission capabilities [7]. The placement problem can be considered as an instance of the cover set problem, known to be NP-hard [8]. It is a problem to determine the most suitable locations from a set of possible locations while satisfying constraints. In the case of the WSN, the constraints are related to energy-efficient activity scheduling, disjoint network recovery, and the design of the communication protocol itself. Besides, assuming a disk-shaped communication range, the placement problem can be addressed as the art gallery problem, also known to be NP-hard [9]. Thus providing further options for possible approaches to be used, such as [10].

Motivated by both fields, we propose a novel connectivity-aware relay placement that addresses specific constraints arising from the practical properties of our communication system used in the DARPA SubT [11]. The low-bandwidth communication is based on relatively low-power radios with a limited range [12]; however, its most important feature is that it does not rely on known or established routing topology. A flood-routing protocol is used, where the relays broadcast all received data. Thus, it is desirable to limit the number of relays within the local connectivity to mitigate packet clashes and media access overheads of the *Carrier Sensing Multiple Access with Collision Avoidance* (CSMA/CA) based protocols [13] of the IEEE 802.11 Distributed Coordinated Function. On the contrary, connectivity redundancy is also desired to improve the reliability of packet delivery.

Further motivated by [10], we leverage a known fixed-radius sensing range to restrict the space to be covered by the communication network, assuming the robot does not need to travel toward obstacles closer than the sensing range in a search mission. The space to be covered is then divided into Open space and Corridor space based on the expected signal propagation model, see Fig. 2. In Open space, a rigid three-connectivity regular tessellation pattern is used since the signal is expected to propagate omnidirectionally up to a specified maximum range of reliable communication. For

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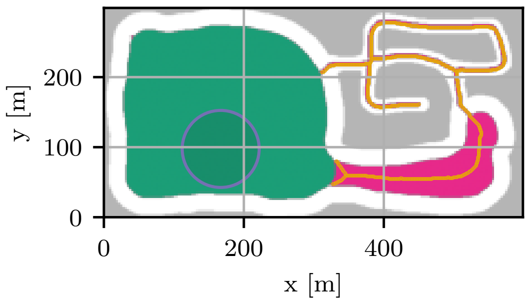


Fig. 2. Classification of the areas into occupied space (gray) and free space (white) that is closer to the obstacles than the sensing range δ_s. Areas to be covered are Open space in green and Corridor space in pink. The *medial axis* is in orange. The communication range δ_c is illustrated in purple.

Corridor space, equidistant relay placement along the *medial axis* with the line-of-sight condition establishes a naïve but conservative placement with a guaranteed minimum degree of connectivity. Finally, the residual uncovered areas are covered greedily. Note that the proposed idea of the placement of the communication modules is demonstrated for available known maps of the static environment to support inspection and patrolling missions. The presented results support that the proposed approach is viable, motivating further research on online deployments with incremental placement rules.

The proposed approach is compared with the existing *Randomized Dual Sampling* (RDS) [14] that is considered a representative reference approach because it allows relatively straightforward adjustment to satisfy relay placement constraints. The reported results show that the proposed method places relays with improved algebraic connectivity and fewer relays than the RDS while still keeping desired network redundancy and coverage.

The rest of the paper is structured as follows. An overview of existing approaches is presented in the following section. The addressed problem is formally introduced in Section III. Section IV summarizes the baseline approach. The proposed method is presented in Section V. The evaluation results are reported in Section VI. Section VII concludes the paper.

II. Related Work

Communication relay placement by mobile robots attracted attention within the DARPA SubT [4], [15]. Despite the overall complexity of the systems developed by the participating teams, the employed placement approaches are relatively basic. The winning team, CERBERUS, relied on a human supervisor to deploy the relays ad-hoc [6]. A reactive automated strategy has been adopted by the team CoSTAR [5]. An attempt to tackle relay placement with a level of algorithmic optimization is demonstrated in [16] using estimating unseen parts of the environment. Note that using the so-far built environment model, the studied relay placement using a known environment map still applies in scenarios where the environment map is built during the mission, such as the DARPA SubT.

Automated relay placement is also studied in [17] based on exploiting signal propagation patterns in straight tunnels. Further, visible-light communication in mobile relays is proposed in [18] based on the identified distance-based signal strength model. Moreover, subterranean communication is crucial as the next step of possible extraterrestrial habitats [19]. However, to the best of the authors' knowledge, precise signal propagation modeling is not widely considered in relay placement because it is challenging [20], [21].

In basic scenarios without obstacles, a simple omnidirectional propagation with the disk-shaped coverage region can be assumed as a sufficient communication model for the defined maximum reliable communication range [22]. Still, in cluttered urban and industrial environments, the signal is deteriorated by reflections and induced multipath propagation or shadowing, which may be tackled either by data-driven models [23] or statistically via known fading models [24]. Furthermore, in constrained corridors, a waveguiding effect takes place, making it possible to employ empirical signal propagation models [25] to exploit communication accessibility by the waveguiding effect eventually. Thus, relay placement approaches should account for signal propagation models ranging from simple disk-shaped modeling to more advanced data-driven methods [12].

A communication protocol is required to propagate the information in a network efficiently [7]. In flat flood routing, each node rebroadcasts the incoming message in a peer-to-peer fashion. Hierarchical concepts may introduce energy efficiency, autonomy, or resilience. However, regardless of the protocol, all nearby active relays share limited channel bandwidth. Hence, the degree of relay connectivity is important because improper placement can easily cause congested areas, resulting in packet collisions during CSMA/CA randomized link access scheme [13]. The hexagonal tesselation pattern might be helpful as it covers the space with the desired degree of connectivity [26], [27]. Thus, it inspires us to use it to cover large open areas where disk-shaped signal propagation modeling is sufficient.

The WNS placement problem is closely related to the studied relay placement. In addition to the area coverage, the latter requires the relays to be mutually in range. Numerous approximations to the otherwise NP-hard cover set problem are employed in computing the sensors' locations in WSNs. In sensor placement, submodularity might be exploited in greedy-like near-optimal placement [28] while exploiting particular communication link properties. Particle Swarm Optimization [29] and Mixed-Integer Linear Program [30] have been utilized to search for a complete relay layout considering the sensor model and communication mode. Geometric approaches have also been proposed based on Voronoi diagrams [31] or spiral pattern [32]. Further, the adaptive centroid-based heuristic [33] or Steiner tree [34] may be used to compute connecting paths for partial network failure. However, these methods perform the placement uniformly without exploiting local topology, which can significantly enhance the placement and reduce the cost.

In the presence of obstacles, the placement problem with mobile robots is also studied as the art gallery problem [35] with randomized sampling-based methods in the polygonal map representation [10], where a straight skeleton can be used to guide connections of the relays [36]. Since the addressed relay placement problem needs to account for defined connectivity constraints, we consider the existing *Randomized Dual Sampling* (RDS) algorithm [14] as a suitable candidate to be adjusted for the studied relay placement.

III. Problem Statement

The relay placement model is studied for occupancy grid-like world domain $\mathcal{W} \subset \mathbb{R}^2$ and occupancy function $F : \mathcal{W} \to \{0, 1\}$, where 0 denotes a spatial element is free of solid obstacles, and free space defined as $\mathcal{F} = \{x \in \mathcal{W} : F(x) = 0\}$. The area of $\mathcal{W}$ required to be communication accessibile is denoted $\mathcal{I} = \{x \in \mathcal{F} : I(x) = 1\}$, where $I : \mathcal{F} \to \{0, 1\}$ determines whether a spatial element is required for coverage, $I(x) = 1$. $\mathcal{W}, \mathcal{F}$, and $\mathcal{I}$ are assumed to be connected spaces. For the studied inspection/patrolling-like missions, the whole $\mathcal{F}$ can be searched by the robot sensor system while traversing only its subset $\mathcal{I} \subset \mathcal{F}$.[1]

The relay placement is to find a minimal set of s relay locations $r_i \in \mathcal{F}$ forming a network $\mathcal{N} = \{r_1, \ldots, r_s\}$ that covers $\mathcal{I}$. A point $x \in \mathcal{I}$ is said to be covered by a relay at $r \in \mathcal{N}$ if x is communication accessible directly from r without any other relay in $\mathcal{N}$. For example, for a disk-shaped communication model, x is covered by r if their mutual Euclidean distance $\|x - r\| < \delta_c$, δ_c being the communication range, and x and r are mutually visible, i.e., $\forall y \in \{r + t(x - r), t \in (0, 1) \subset \mathbb{R}\} : y \in \mathcal{F}$. It is sufficient that each $x \in \mathcal{I}$ is covered by at least one relay.

Further, the resulting relay network is required to be connected and redundant. Two relays $r_i, r_j \in \mathcal{N}$ are considered neighbors if they are mutually directly communication accessible; the number of neighbors of a relay r is denoted as the relay degree d. The network $\mathcal{N}$ is considered connected if a progression of neighbors exists between every relay pair. $\mathcal{N}$ is considered redundant if any two relays in the network share at least two different neighbor progressions; the more distinct progressions, the more redundant the network is. Finally, the relay degrees need to be minimal to mitigate physical channel congestion or packet clashes. Thus, the network redundancy and relay degrees shall be balanced.

A. Relay Placement Quality Indicators

The relay placement method is to minimize the number of relays s of $\mathcal{N}$ while balancing its redundancy and constraining the relay degrees. Therefore, we examine the quality of the relay placement $\mathcal{N}$ using the following quality indicators.

- The number of relays $s = |\mathcal{N}|$.
- Percentage of $\mathcal{I}$ covered by $\mathcal{N}$ denoted `Coverage%`, determined as a union of area coverage $\mathcal{S}(r)$ by each $r \in \mathcal{N}$,
$$\texttt{Coverage\%} = \frac{|\bigcup_{r \in \mathcal{N}} \mathcal{S}(r)|}{|\mathcal{I}|}.$$
- Average area covered by a relay $\texttt{ApR} = \frac{\texttt{Coverage\%}}{s}$.
- Network redundancy `FiV` quantified as the algebraic (Fiedler) graph value [37].
- Average relay connectivity degree $\overline{\texttt{RCD}}$ and the accompanied standard deviation $\sigma_{\texttt{RCD}}$.

[1] The way how $\mathcal{I}$ is determined is a part of the proposed method.

IV. Baseline Approach

The existing *Randomized Dual Sampling* (RDS) [14] is selected as the baseline approach among existing approaches, such as [28], [29], [32]. We opt for the RDS because it allows a straightforward generalization for relay placement constraints. The original approach is based on the sampling boundary of the area to be covered, and within the sampled disk-shaped coverage, additional disk-shaped samples are made, and the best covering one is selected as the placement location. We adopted the sampling for the relay placement with additional constraints of inter-relay connectivity and coverage. The algorithm is summarized in Algorithm 1 and works as follows.

Algorithm 1: Randomized Dual Sampling (RDS)

Input: $\mathcal{I}$ – Space of interest to be covered.
Parameter: δ_c – Communication range.
Parameter: m – Number of random samples.
Parameter: d – Desired connectivity degree.
Output: $\mathcal{N}$ – Set of relay locations.

1 $\mathcal{N} = \emptyset$
2 $U \leftarrow \mathcal{I}$ // Initialize uncovered space
3 **while** $|U| > 0$ **do**
4 $\partial U \leftarrow$ `boundaryOf`(U) // get boundary
5 $p \leftarrow$ `randomPoint`(∂U)
6 $V \leftarrow$ `visible`(p, U, δ_c) // disk-shaped vis.
7 $C \leftarrow$ `randomSubset`(V, m) // candidates
8 $\hat{C} \leftarrow$ `connectivityMatch`(C, N, d)
9 $c^\star \leftarrow \arg\max_{c \in \hat{C}} |$`visible`$(c, U, \delta_c)|$
10 $\mathcal{N} \leftarrow \mathcal{N} \cup \{c^\star\}$
11 $U \leftarrow U \setminus$ `visible`$(c^\star, U, \delta_c)$
12 **end**

The RDS is an iterative procedure that incrementally covers not yet covered area U of the given $\mathcal{I}$. At each iteration, a relay location is determined using dual sampling. First, a random point p on the boundary of ∂U is determined. A disk-shaped communication model is applied from p using visibility on the grid [38] and communication range δ_c. In the visibility area V, m random locations C are sampled as candidate locations. Since we aim to achieve the desired relay connectivity degree d, locations C are examined for the connectivity degree to match the desired degree d. If such locations are not found, the locations with the closest degree to d are selected to form $\hat{C}$ (Algorithm 1, Line 8). The set $\hat{C}$ is the main adjustment of the original RDS for sensor placement to the addressed relay placement. The location $c^\star \in \hat{C}$ with the largest coverage of U is then added to the set of relay locations $\mathcal{N}$, and U is updated (Algorithm 1, Lines 9 to 11). The procedure is repeated until U is covered.

V. Proposed Method

The proposed method computes a static relay placement following the idea of the Boundary Placement [10] by dividing $\mathcal{F}$ into parts covered by individual procedures and avoiding placement close to obstacles. The proposed method thus consists of creating $\mathcal{I}$ from $\mathcal{F}$ and three phases corresponding to covering $\mathcal{I}$ divided into Open space, Corridor space, and remaining parts, see Fig. 2. In Phase I, Open space is covered with a regular pattern with the asserted connectivity. We consider a hexagonal pattern with $d = 3$, which can be straightforwardly enhanced to $d = 6$ using triangular patterns. In Phase II, Corridor space is covered in a linear chain of relays by a greedy flood-fill strategy with the added line-of-sight constraint. Finally, possible remaining areas are greedily covered in Phase III. The environment is thus classified into regions where specific signal propagation characteristics can be exploited. The procedure is summarized in Algorithm 2 and detailed in the rest of the section.

Algorithm 2: Proposed Relay Placement

Input: $\mathcal{I}$ – Space of interest to be covered.
Parameter: δ_c – Communication range.
Parameter: d_o – Open space node degree.
Parameter: d_c – Corridor space node degree.
Output: $\mathcal{N}$ – Set of relay locations.

▷ Phase I – *Cover Open space*
$\hat{F} \leftarrow$ `shrink(`$\mathcal{I}, \delta_c$`)` `// Reduced free space.`
$B \leftarrow$`grow(`$\hat{F}, \delta_c$`)` `// Determine Open space.`
$N_B \leftarrow$`tesselator(`B, δ_c, d_o`)`
▷ Phase II – *Cover Corridor space*
$C \leftarrow$ `corridors(`$\mathcal{F}, B, \delta_c$`)`
$N_C \leftarrow$`coverAlongSkel(`C, N_B, δ_c, d_c`)`
▷ Phase III – *Cover not yet covered parts*
$R \leftarrow \mathcal{I} \setminus$`coverage(`$N_B \cup N_C$`)`
$N_G \leftarrow$ `coverGreedy(`R`)`
$N \leftarrow N_B \cup N_C \cup N_G$

Determination of the area to be covered: $\mathcal{I}$ is determined by shrinking $\mathcal{F}$ by the robot sensing range δ_s. Possibly closed corridors are represented in $\mathcal{I}$ as a *medial axis* of $\mathcal{F}$. Hence, $\mathcal{I}$ consists of a free space reduced by space that the robot does not need to traverse during inspection missions, see Fig. 2.

Phase I – Open space Coverage

The part of the environment to be communication accessible $\mathcal{I}$ is divided into Open space and Corridor space. Open space identifies areas where signal propagates omnidirectionally without obstacle-induced deteriorations, where it is possible to reckon with the maximum communication range. A gap-filling technique is used to detect Open space [39]. First, reduced free space $\hat{\mathcal{F}}$ is computed by shrinking $\mathcal{I}$ by δ_c using distance transform and then growing it back. The yield is a part of $\mathcal{I}$ corresponding to open space of at least $2\delta_c$ diameter, as the space near obstacles is closed.

Open space, being void of free-standing obstacles, allows strong assumptions about signal propagation with the communication range δ_c. The relays can be thus placed using a hexagonal tesselation lattice with guaranteed connectivity. Placing relays at the vertices of the hexagonal grid with a side δ_c long, each relay shares exactly 3 other neighbors. It holds if the used radio cannot directly communicate to

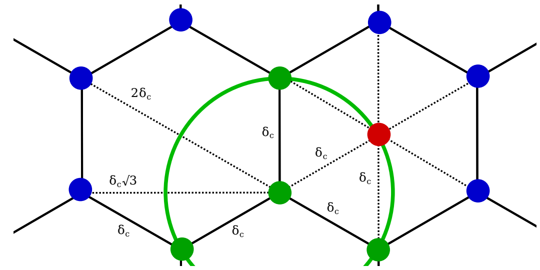


Fig. 3. Tesselation pattern used for covering Open space. Relay locations are depicted as small disks. The green disks denote a triple of relays connected with a communication range δ_c illustrated as the green circle. The blue disks depict other relays in the resulting network $\mathcal{N}$. The red disk is a possible location at the center of the hexagon that can be used for the resulting 6-connected network.

the second nearest neighbor at a distance $\delta_c\sqrt{3}$. Besides, 6-connectivity with the same margin can be achieved using a triangular lattice that can be straightforwardly created from the hexagonal grid as illustrated in Fig. 3. We evaluated both lattices as $d_o \in \{3, 6\}$ for the relay placement quality.

In practice, selecting $\delta_c = 55\,\mathrm{m}$ places the second nearest vertex around $95\,\mathrm{m}$ away. Partial redundancy of the network is asserted by requiring the Open space to be at least $4\delta_c$-wide. Hence, the lattice may be freely rotated while each vertex has a neighbor part of at least one full hexagon or triangle. One can easily subdivide the pattern in the need for denser connectivity. Note that a rectangular grid with 4-connectivity is also possible uniform tesselation.

Phase II – Corridor space Coverage

Once Open space is covered, Corridor space is determined as the remaining space of interest to connect possibly disjoint Open space and further cover narrow corridors. The space is covered using the greedy flood fill strategy along its *medial axis* curves [39] that maximizes the distance to obstacles, for example, using robust λ-*medial axis* [40]. First, a relay is placed at starting point that is selected as any grid cell among the highest-order *medial axis* junctions. The junction order of the grid cell is the count of the neighbor part of the *medial axis*. However, cells with two *medial axis* neighbors are not considered a junction, but cells with a single neighbor are considered the lowest-order junction.

The *Breadth First Search* (BFS) algorithm considering only the *medial axis* is executed from the starting point. Each visited cell is checked on the distance and line-of-sight condition to the relay placed at the most recent ancestral (in the BFS traversal sense) cell. The linear connectivity degree $d \in \{2, 4, 6, \dots\}$ is established by setting the deployment distance as $\frac{2\delta_c}{d}$. If the two criteria are violated, the relay is placed to the current cells's BFS traversal parent. If a cell with no possible BFS expansion is available, the nearest relay from Phase I is found and connected with a straight line segment if both are directly visible. A separate relay degree d_c is utilized for the Corridor space coverage. In practice, the signal propagation in narrow corridors can be very complex on its own [21], [25]. The advantage of the proposed method is its ability to exploit such propagation models instead of the line-of-sight or range conditions.

Phase III – Remaining Parts Coverage

The placement from Phase I provides coverage without internal holes; only possible coverage holes might be found at the boundary. Corridor space coverage guided by the *medial axis* is used in areas less than $4\delta_c$ wide. Thus, isolated strips, up to δ_c wide, might still be left for the coverage. Such leftover remaining parts form the area to be covered. Since possibly suboptimal deployment can have only a local impact, the parts are covered greedily. The linear, equally spaced relay chain asserts the connectivity to the nearest *medial axis* or open space relay.

VI. RESULTS

The presented solutions to the introduced relay placement problem have been empirically evaluated. We hypothesize that classifying the environment in the proposed methods to use respective specific coverage patterns yields the same spatial coverage with fewer relays while improving the connectivity metric than a solution provided by the baseline RDS method. The performance of the methods is evaluated using the indicators as defined in Section III-A.

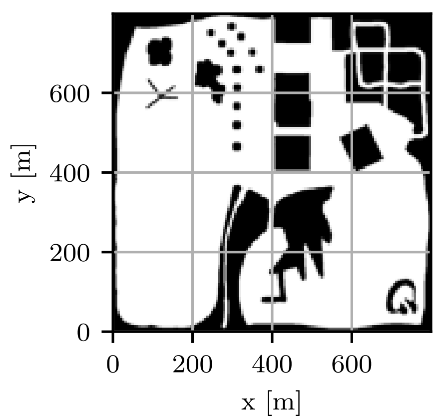


Fig. 4. Benchmark map, where free space is white, and obstacles are black.

The evaluation is performed for a benchmark map depicted in Fig. 4. It includes different topologies, so only a single map is examined. The map represents a $800\,\mathrm{m} \times 800\,\mathrm{m}$ large area with the grid cell size of $1\,\mathrm{m}$. The map is designed to include the following topologies found in robotic missions.

- The lower left portion contains a large open space.
- The central bottom section contains a long corridor
- The right bottom part includes an artificial bug trap.
- The upper right section features a corridor maze commonly found in room-and-pillar mines.
- The upper left part contains an assortment of obstacles.

The RDS baseline and the proposed methods have been evaluated for three different parameterizations. The RDS has been examined for three relay connectivity degrees $d \in \{3, 4, 5\}$. Because the RDS is a stochastic algorithm, the reported values are averaged over 20 runs per setup. The proposed method has been examined for hexagonal tessellations pattern providing 3-connectivity of Open space and with $d \in \{2, 4\}$ connectivity used in covering Corridor space. Besides, a triangular pattern (using centers of the hexagonal cells) is used for 6-connectivity of covering Open space and $d = 4$ used in covering Corridor space. The sensor range is set to $\delta_s = 25\,\mathrm{m}$ and the communication range to $\delta_c = 55\,\mathrm{m}$. The resulting coverages are summarized in Table I. The computational time of non-optimized implementation in Python is listed in the column $\mathrm{T}_{\mathrm{CPU}}$ for the Intel® Core™ i7-8550U. Examples of found placements are depicted in Fig. 6.

Fig. 5. Illustration of the RDS coverage progress (from left to right). The space is gradually covered (green), leaving temporary holes (red) covered in the following iterations.

A. Discussion

The evaluation results indicate that the proposed environment classification to Open space and Corridor space helps with the communication relay placement. The proposed method requires fewer relays to cover the whole $\mathcal{I}$ while improving the Algebraic (Fiedler) Connectivity `FiV` [37]. The network redundancy is asserted with a connectivity degree around $d = 4$ to 6, providing multiple routes for packet distribution. Channel congestion and packet clashes can impact communication reliability, which would need stochastic simulation that is out of the paper's scope.

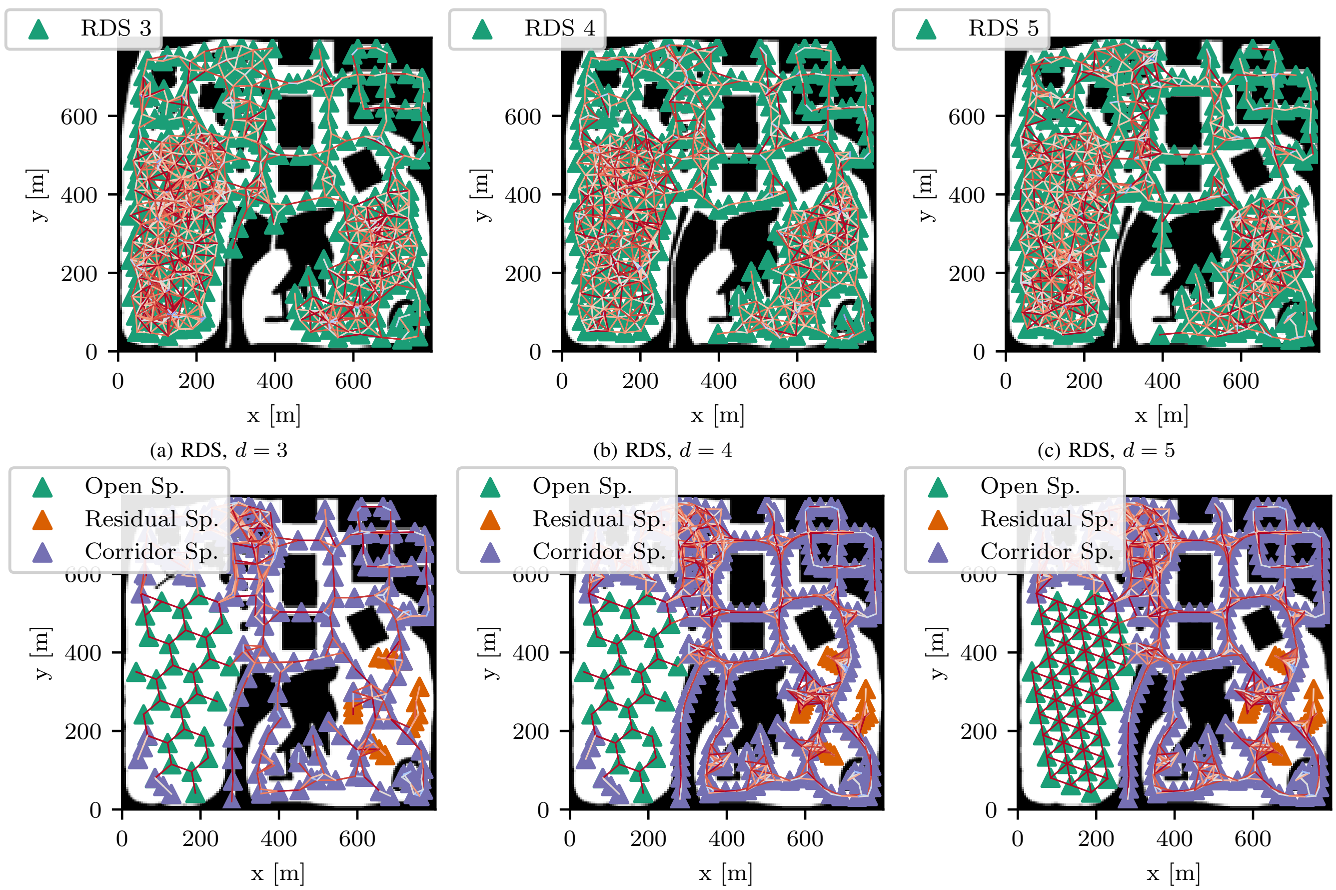
The proposed Open space coverage with triangular or hexagonal patterns covers the required space close to the required connectivity. The regular pattern fixes the inherent downside of the RDS that greedily leaves temporary local coverage gaps, as shown in Fig. 5. The holes are gradually covered with additional relays, resulting in over-deployment and possible network degradation by cross-talks. Such property contradicts the RDS nature, which always tries to extend the coverage without any "unnecessary" overlap.

Further, guiding the proposed deployment by the medial axis in the corridor space introduces guaranteed connectivity, whereas the RDS may fail to extend the network to constrained spaces, as apparent from Fig. 6. In the corridor, only an even connectivity degree is feasible. Sparse coverage gaps at the area border, see the right bottom part of Fig. 6e, are considered residual space covered greedily without significantly impacting the overall network connectivity.

The proposed methods provide generative rules that are implementable onboard with only partial environment knowledge. Hence, a local map can be sufficient for partial relay deployment. The algorithm can operate incrementally since both the obstacle inflation and medial axis operations can be performed locally, and the lattice requires only knowledge of near neighbors to be continued. Thus, the regular-interval deployment along the medial axis in narrow passages or the fixed-pattern tessellation in a wide space can be employed dynamically. Therefore, the method can be deployed to explore network-constrained environments where communication relay placement needs to be determined online.

TABLE I
SUMMARY OF THE ACHIEVED RESULTS.

Method name	s [-]	coverage [%]	ApR [m^2]	FiV [-]	$\overline{\text{RCD}}$ [-]	σ_{RCD} [-]	T_{CPU} [s]
RDS, $d = 3$	518	91.9	589	0.0056	13.4	7.7	30.1
RDS, $d = 4$	465	91.4	594	0.0053	11.2	6.4	262.6
RDS, $d = 5$	532	98.5	541	0.0048	11.5	4.5	256.0
Prop., hexagonal ($d_o = 3$), corridor linear coverage $d_c = 2$	241	99.8	1 100	0.0084	4.3	2.4	13.1
Prop., hexagonal ($d_o = 3$), corridor linear coverage $d_c = 4$	359	99.8	720	0.0135	6.6	3.1	15.0
Prop., triangular ($d_o = 6$), corridor linear coverage $d_c = 4$	367	100.0	700	0.0156	6.8	2.8	15.4



(a) RDS, $d = 3$ (b) RDS, $d = 4$ (c) RDS, $d = 5$

(d) Proposed, hexagonal pattern ($d = 3$), linear coverage $d = 2$ (e) Proposed, hexagonal pattern ($d = 3$), linear coverage $d = 4$ (f) Proposed, triangular pattern ($d = 6$), linear coverage $d = 4$

Fig. 6. Example of the found coverages.

VII. CONCLUSION

A novel relay placement method is proposed motivated by mobile robotics search and inspection missions in communication-denied environments. It explicitly classifies the environment into Open space and Corridor space with dedicated covering strategies. Based on the performance comparison with the existing RDS schema adjusted to address the relay placement, the proposed three-phase algorithm provides significantly better solutions than the RDS. Hence, the proposed placement strategy is vital and further motivates the deployment of the method in exploration-like missions using multiple robots. Further, we aim to improve the placement in the residual areas, formulate the method for dynamic obstacles, and employ specific signal propagation models for corridors and open spaces coverage.

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